PYTHAGORAS

And the Mathematization of the World

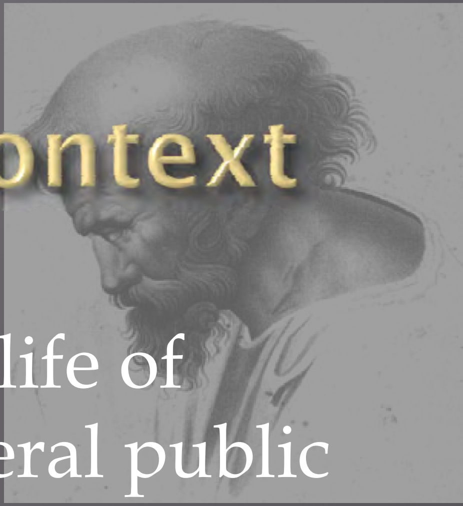
Pythagoras – Historical Context

- was born on the island of Samos in the Aegean Sea at around 560 BC

Pythagoras – Historical Context

- He founded a religious cult that came to be known as the Pythagorean Order
- As a cult, they:
  - Were vegetarians
  - Did not consume alcohol
  - Had no lasting possessions.
  - Believed in a soul that could exit the body either temporarily or permanently, and move into another human's body.
  - Believed everything in the known universe could be explained by numbers

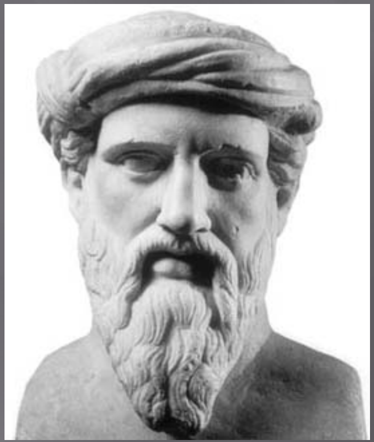
Pythagoras – Historical Context



- These beliefs, especially the belief in a life of poverty, contradicted those of the general public and caused the Pythagoreans to be chased off of the island of Samos
- Pythagoras settled in Croton, Italy, where he established his School of Pythagorean Order.

Pythagoras – Historical Context

- It was at this school where most of his amazing mathematical discoveries took place.
- His discoveries, however, may have been the discoveries of his followers, since Pythagoras himself did not leave behind any written record.



Pythagoras – Historical Context

- Some of his doctrines opposed democracy by advocating aristocratic rule, which inevitably led to his being exiled from Croton.
- He then moved to Metapontion on the Gulf of Tarento

- He died around 500 BC

PYTHAGORAS

His Contribution to Number Theory

Figurate Numbers

- The Pythagoreans were quite intrigued by *figurate* numbers, which are numbers made from counting in patterns

- The two most common types of these numbers are *triangular* numbers and *square* numbers

Triangular Numbers

- Those numbers that have an arrangement yielding a triangle are known as *triangular* numbers.
- To obtain the next triangular number, add another row
- The first 5 triangular numbers are : 1, 3, 6, 10, 15

Square Numbers

- Those numbers that have an arrangement yielding a square are known as *square* numbers
- The first five square numbers are : 1, 4, 9, 16, 25

Perfect Numbers

- A *perfect* number is a number that is equal to the sum of its *proper* divisors (divisors that exclude the number itself)
- The first five perfect numbers are:
  - 1 : 1 = 1
  - **6** : 1 + 2 + 3 = 6
  - 28 : 1 + 2 + 4 + 7 + 14 = 28
  - 496 : 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496
  - 8128 : 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064 = **8128**

Amicable Numbers

- A pair of numbers are said to be *amicable* if each number is equal to the sum of the other numbers divisors.
- The simplest example of an amicable pair is the pair 220 and 284:
  - **220**: 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = **284**
  - **284**: 1 + 2 + 4 + 71 + 142 = **220**

PYTHAGORAS

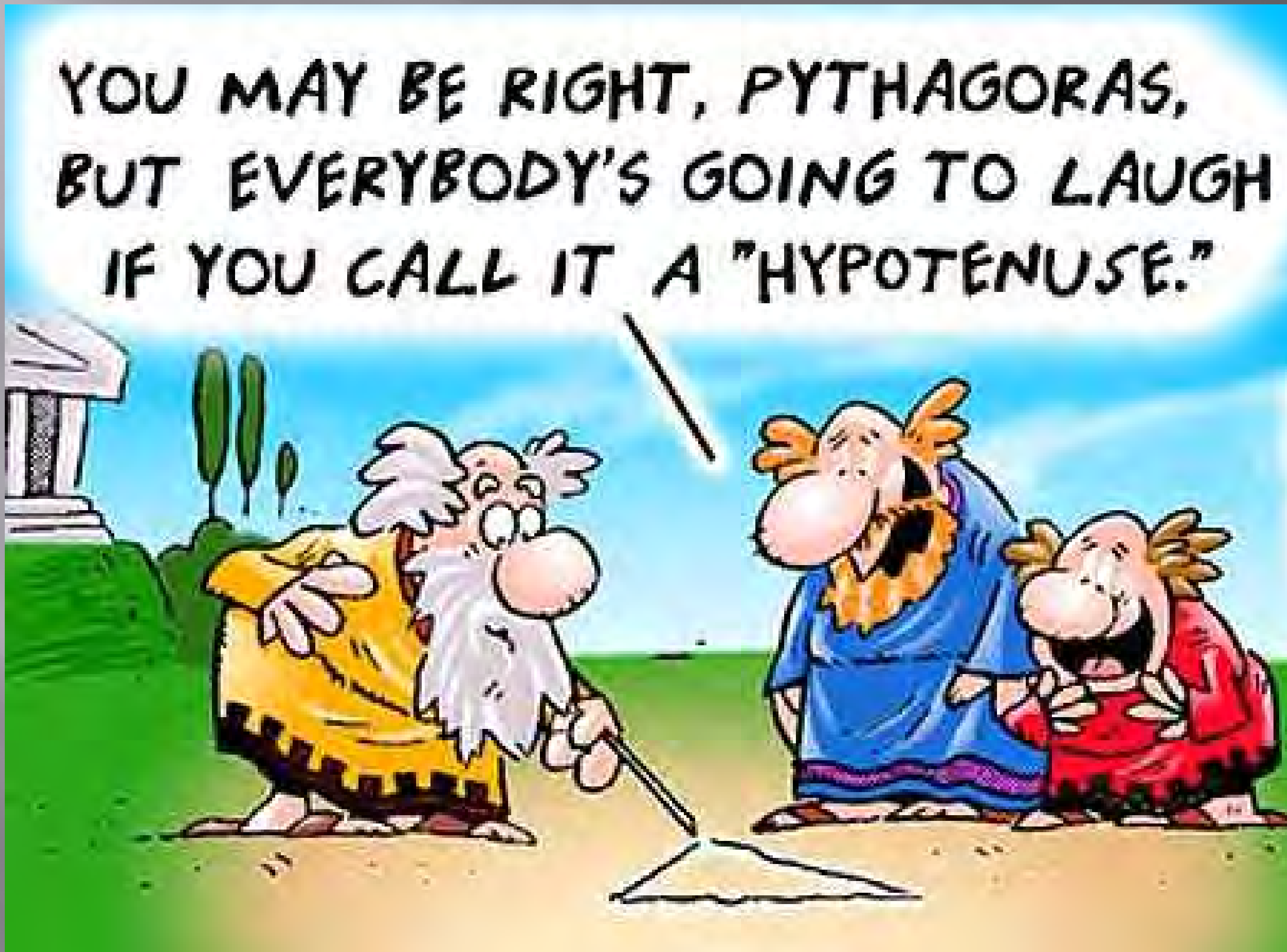
His Contribution to Harmonics

Pythagoras & Harmonics

- Pythagoras discovered that if the length of a string was cut in half from 12 units long to 6 units long, the note would increase by one octave
- He also discovered that if the string's length was reduced from 12 units to 8 units the note produced would sound a fifth above the original note
- He discovered that these three notes played together made a sound very pleasing to the human ear

Pythagoras & Harmonics

- Based on these discoveries, he concluded that the numbers 6, 8, and 12 were in a harmonic progression
- Pythagoras decided that a cube, which has 6 faces, 8 corners, and 12 edges, was in perfect geometric harmony
- He later went on to discover the multiples required to make every sound on our current musical scale


YOU MAY BE RIGHT, PYTHAGORAS, BUT EVERYBODY'S GOING TO LAUGH IF YOU CALL IT A "HYPOTENUSE."

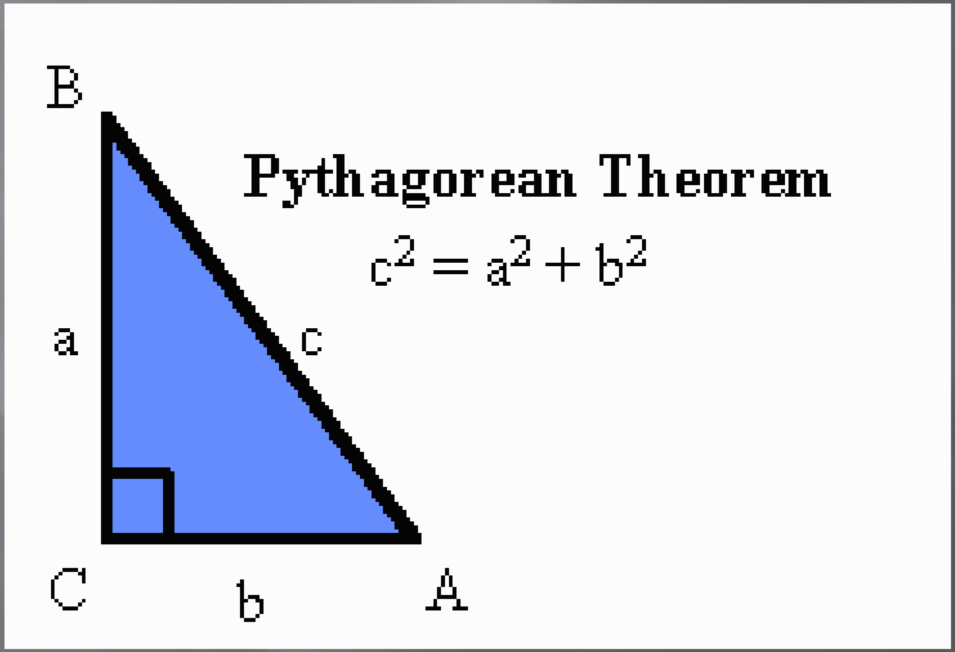
The Scarecrow *doesn't* have a Brain

PYTHAGORAS

His Famous Theorem

The Pythagorean Theorem

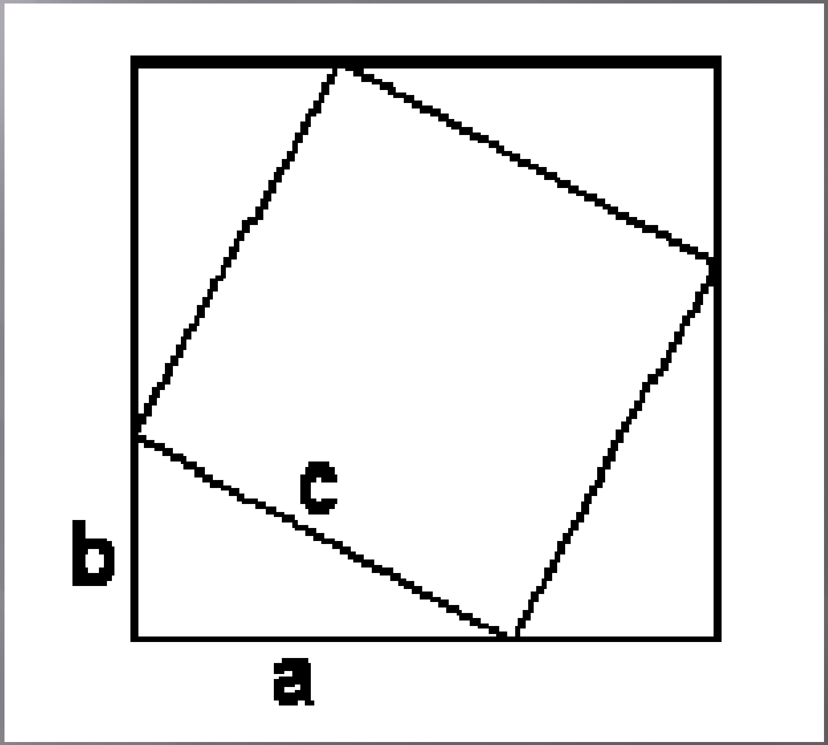
- Given a right triangle whose sides are *a, b,* and *c,* the relationship between these sides can be described by the following formula:



The Pythagorean Theorem

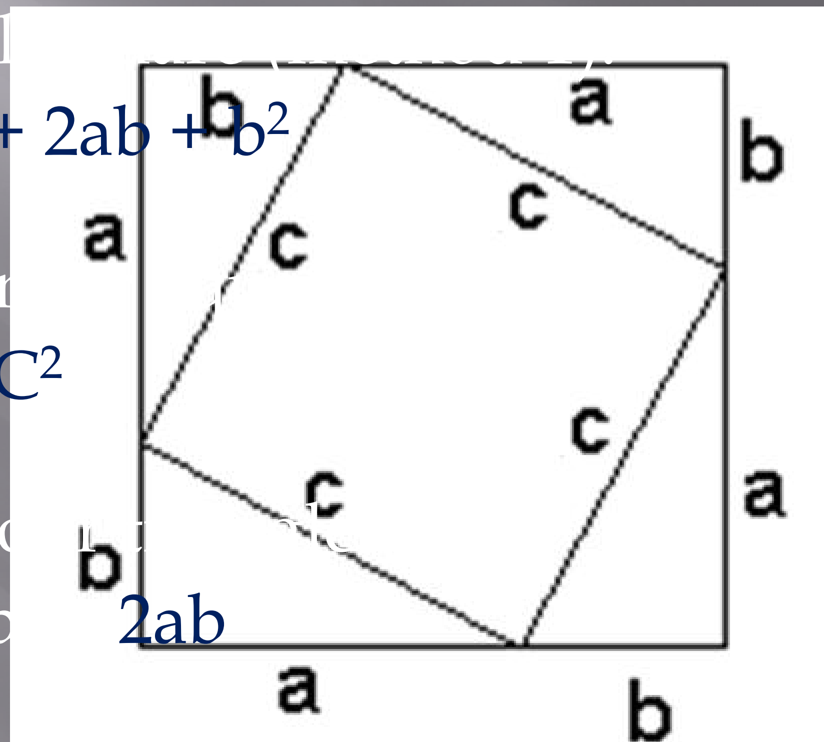
- The theorem itself was well-known to the Egyptians thousands of years earlier, the only thing Pythagoras did was popularize it and possibly give a proof for it.

On Possible Proof



Area of Total

$(a + b)^2 = a^2 + 2ab + b^2$



Area of the i

C times C = C^2

Area of the fo

4 times ½ (ab 2ab

Area of Total Square (method 2):

Area of inner square + Area of triangles

Area of Total Square (method 2):

Area of inner square + Area of triangles

$$c^2 + 2ab$$

Since we found the total area two different ways, both ways must give the same answer, that is, they will be equal.

$$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$$

$$\boxed{a^2 + b^2 = c^2}$$