The 3 Unsolved Problems of Greek Mathematics

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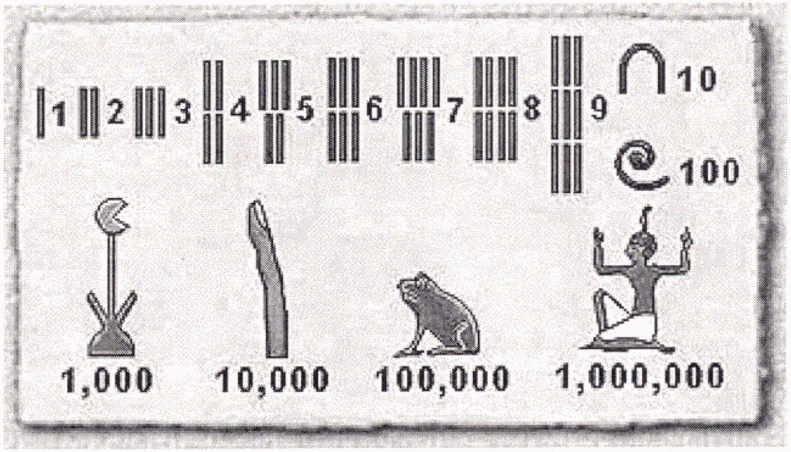
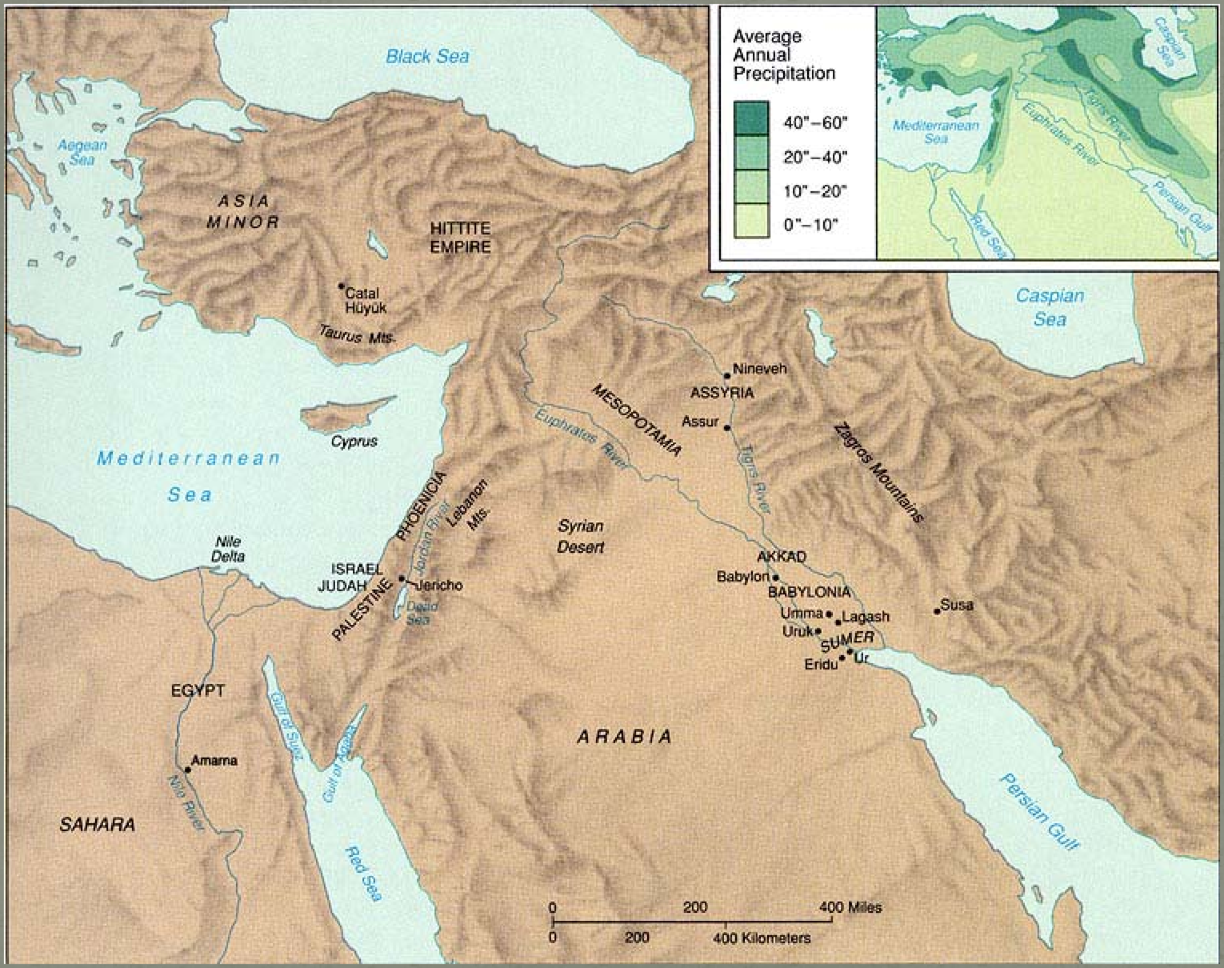
Historical Context - Numbers

- By around 2000 BC, Geometry was developed further by the Babylonians who conquered the Sumerians.
- By around 2000 BC, Rational and Irrational numbers were used by the Babylonians who conquered the Sumerians.
- The Egyptians further developed math but, like the Sumerians, were concerned with applied math.



http://saxakali.com/COLOR_ASP/historymaf5.htm

MAP: Sumeria/Babylon/Mesopotamia/Egypt

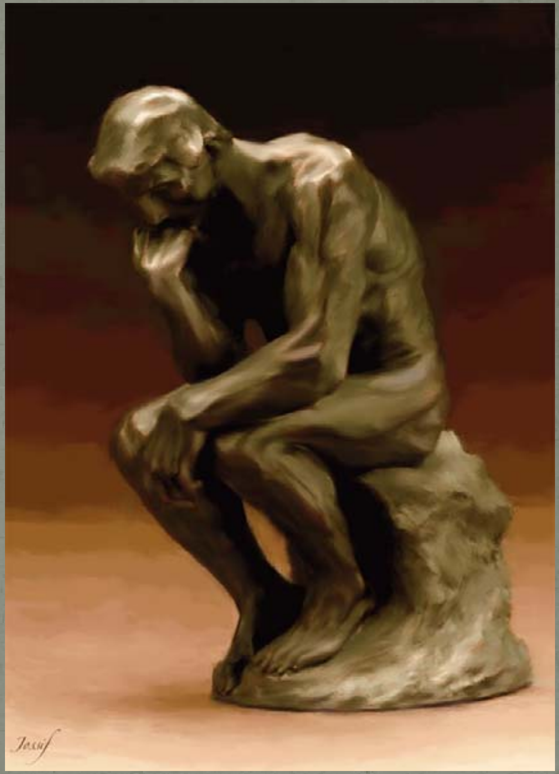


Historical Context – Speculative Thinking

- The ancient Greeks were immigrants and all their science in origin was dependant on traditions from more ancient civilizations; the Sumerians, Babylonians, Egyptians and Mesopotamians. This, the Greeks themselves insisted.
- The thing that set the ancient Greeks apart was that they were the first to search for understanding in math not inspired by religion or its practical applications.

Historical Context - Origins

- With this new type of philosophical thinking that promoted reasoning, it wasn't before long that a mathematical revolution occurred and thus math, especially geometry, was further developed than any of the other sciences during the time of the ancient Greeks.

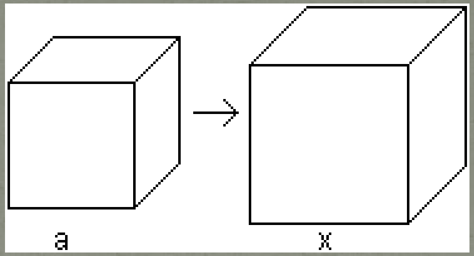


Historical Context - Importance of Geometry

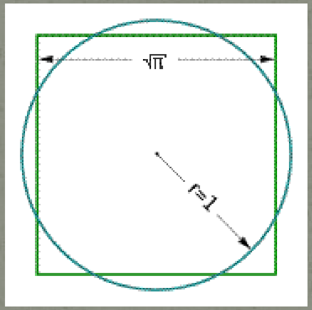
- Thales, familiar with Egyptian mathematics was considered to be the founder of geometry.
- Pythagoras emphasized further the importance of math and thought that the world could be explained through numbers.
- Further, geometry was viewed by Plato as the basis for the study of any science and as such it was during this time that the three problems of antiquity came to be.

The Three Problems of Antiquity

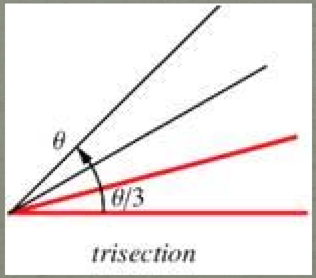
- Doubling the cube



- Squaring the circle



- Trisecting the angle



Historical Context - The Origins

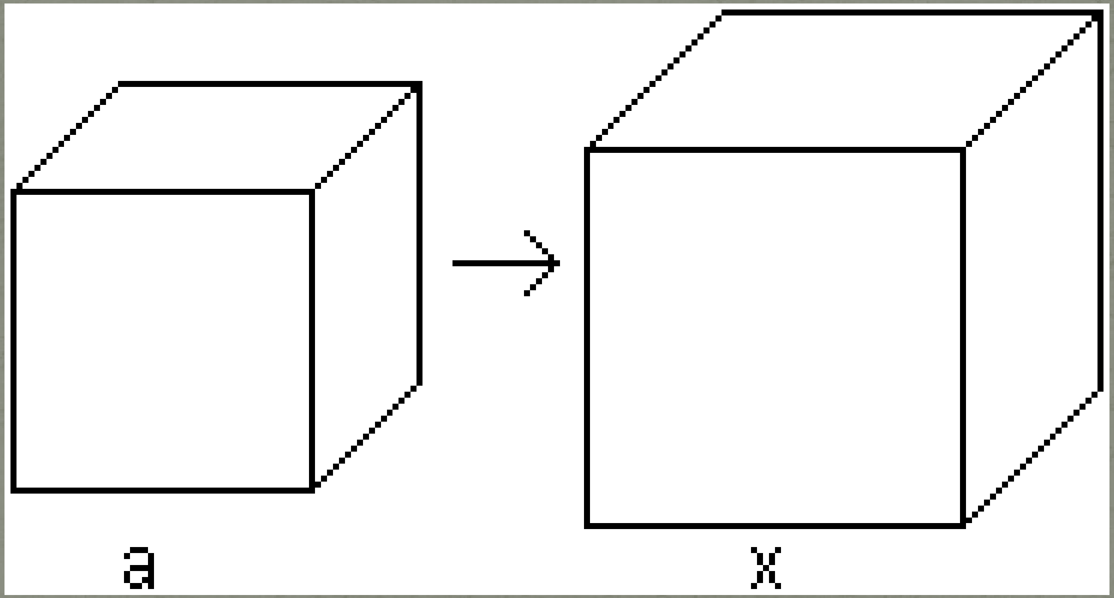
- The origins of the three classic problems are unsure but for the problem of doubling the cube it was said that in 430 BC a great plague struck Athens and its great leader was killed.
- The Athenians sought guidance from the Oracle at Delos who said that the god Apollo was angry and if his cubical altar was doubled in size the plaque would be abolished.
- The Athenians built a new altar doubling the width, height, and depth.
- The plague worsened so they went to the Oracle and found that Apollo wanted the *volume* doubled whereas the Athenians had octupled it.

Historical Context - The Origins

- The plague continued until 423 BC; however this problem was not solved before the plague ended.
- It is likely that the first classic problem was squaring the circle. Then arose the trisection problem.
- These became the Three Unsolved Problems of Greek Mathematics.

The Delian Problem

- ***Otherwise known as Cube Duplication***
- Problem: Double the volume of a given cube.



The Delian Problem: Restrictions

- It is important to know that the method of solution to these problems was extremely restricted by Plato; only an unmarked straight edge and a collapsible compass were available.
- Plato also set up a requirement that the solution would count <u>only</u> if it were accomplished by a geometric construction using these restrictions.

The Delian Problem - Restrictions

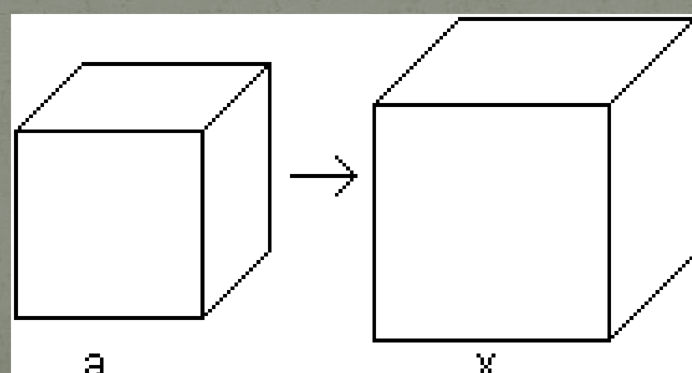
- By means of these two tools, one was restricted to only five mathematical operations (to be performed on rational numbers): addition, subtraction, multiplication, division, and taking square roots.

The Delian Problem – Restricted Sol'n

- Restricted Solution: This requires the construction of a line whose length is the cube root of two and this was impossible to construct under Plato's requirements and so the solution was no solution and was unsolvable for the ancient Greeks.

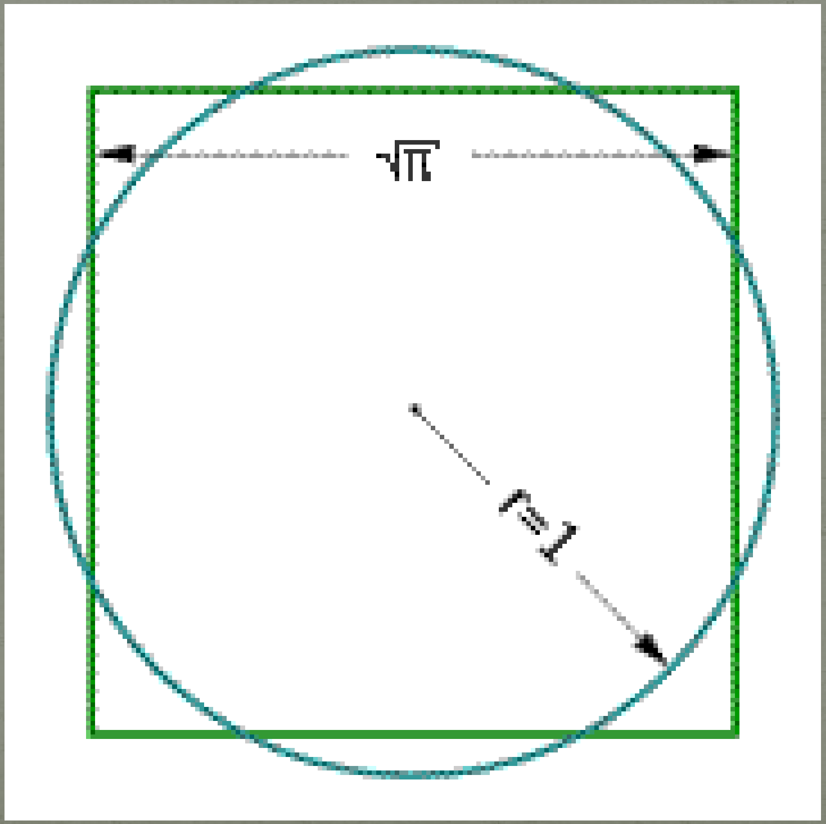
The Delian Problem – Unrestricted Sol'n

- Unrestricted Solution: Today it is easy to solve this problem, it is just an algebraic equation: $a^3=2x^3$ where a is the side length of an arbitrary cube and x is the side length of the new doubled cube as shown in the diagram. With a little algebraic m $x=\sqrt[3]{2}\,a$ ation we get: which is the side length of the original cube with its volume doubled.



Squaring the Circle

- Problem: Construct a square with the same area as a given circle with radius, r.



Types of Numbers

- Before we begin the solution Let's talk about the types of numbers
- Numbers in general (infinite amount of numbers)
- Rational
- Irrational
- Transcendental

Squaring the Circle - Restricted Solution

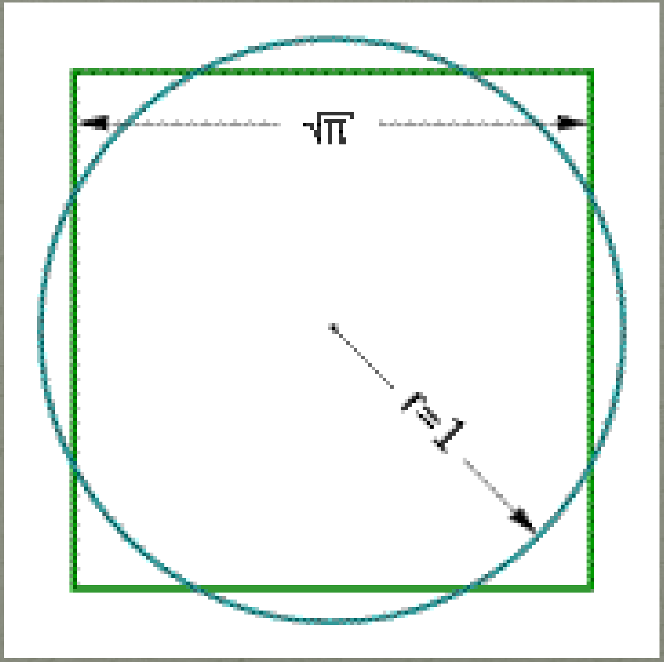
- Restricted Solution: The area of a circle was generalized to be equal to its radius squared multiplied by pi. Pi, however, was not defined exactly which presented a problem: pi is irrational, that is to say it cannot be expressed as a ratio of two rational numbers.

Squaring the Circle - Restricted Solution

- Having known the area of a circle and a square, a simple relationship was noticed: the length of the side of the sought after square is the radius of the given circle multiplied by a factor of the square root of pi.
- As a result of Plato's restrictions, a solution was (and still is) impossible by geometric means.
- Archimedes, amongst many others, toiled over this problem, not realizing pi is a transcendental number.

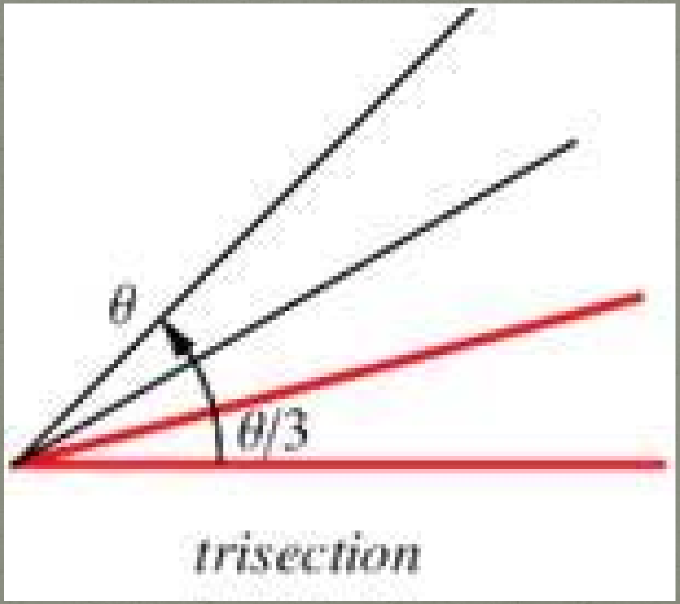
Squaring the Circle – Unrestricted Sol'n

- Unrestricted Solution: For a square to have the area of a circle, its side needs to be the square root of pi times the circle's radius.



Trisecting the Angle

- Problem: Find an angle which measures exactly one third of a given arbitrary angle.



Trisecting the Angle – Restricted Sol'n

- Restricted Solution: The ancient Greeks, having the capability of bisecting angles and trisecting line segments, had difficulties dividing an arbitrary angle into three equal parts.

Trisecting the Angle – Restricted Sol'n

- Granted, the Greeks were capable of trisecting a select few special angles like 90 and 180 degrees, a general method could not be formulated.

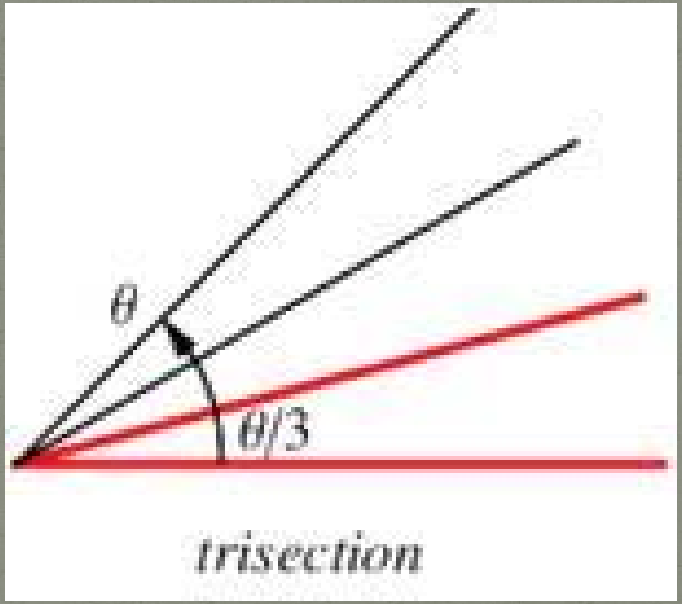
- Trisecting the Angle – Demo
- Trisect 90 Degrees
- Trisecting 180 Degrees

Trisecting the Angle – Restricted Sol'n

- Though Hippocrates and Archimedes (and a few others) did find a way to trisect an angle, it was not within Greek construction under Plato's confines.
- In 1837, the German mathematician Wantzel algebraically proved this problem was impossible.

Trisecting the Angle – Unrestricted Sol'n

- Unrestricted Solution: Algebraically divide the original angle by three. Using a protractor, this is a simple task.



Solutions – Historical Context

- It was only until the 19th century that it was proven that no solution was a solution to the classic problems under Plato's restrictions.
- Using only geometry a circle cannot be squared because it involves the transcendental number π, proven by another German mathematician, Lindemann.
- The cube cannot be doubled because it is impossible to make a line the length of $\sqrt[3]{2}$ under the restrictions.

Solutions – Historical Context

- Archimedes discovered a solution to trisecting an angle; however his solution involved a marked straight edge – which goes against the set restrictions.
- It was not until 1882 that it was proven the problem was unsolvable under Plato's restrictions – also achieved by Lindemann.

Implications for scientific literacy

- There are two important realizations that arose from these three easily understood, but unsolvable problems of antiquity.
- The first is the acceptance of the scientific community that no solution is itself a solution.
- Realizing the truth is much easier than justifying the truth.

Implications for teaching of science

- The simplicity of the problems of antiquity easily captures the interest of the inquisitive mind and in history this encouraged rational thinking and the search for understanding which led to the discovery of new branches of mathematics.
- Today with our knowledge of all the different branches of mathematics these problems can be solved within minutes.

Implications for teaching of science

- Thus, the second important result is the appreciation of all branches of mathematics such as algebra, calculus and geometry.
- The fact that the ancient Greeks were limited to geometry and were not able to solve the problems emphasizes today that all branches of math are inter-related and important to understanding the world around us.

Implications

- Today we know that these classics can be solved easily using other branches of math which are just as important as the branch of Geometry.
- Another important thing to note is that although it may be trivial to us that "no solution" is a solution, it was not so during the time of the Ancient Greeks.
- Plato once said, "…if the problem can be made with geometry, the problem can be solved by geometry…"