The Physics of Equestrian Show Jumping

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This article discusses the kinematics and dynamics of equestrian show jumping. For some time I have attended a series of show jumping events at Spruce Meadows, an international equestrian center near Calgary, Alberta, often referred to as the "Wimbledon of equestrian jumping." I have always had a desire to write an article such as this one, but when I searched the Internet for information and looked at YouTube presentations, I could only find simplistic references to Newton's laws and the conservation of mechanical energy principle. Nowhere could I find detailed calculations. On the other hand, there were several biomechanical articles with empirical reports of the results of kinetic and dynamic investigations of show jumping using high-speed digital cameras and force plates. They summarize their results in tables that give information about the motion of a horse jumping over high fences (1.40 m) and the magnitudes of the forces encountered when landing. However, they do not describe the physics of these results.

In September of 2012 I went to see some of the jumping events at Spruce Meadows. I wanted to see if it was possible to establish a context, imbedded in a good storyline, that would interest many students. I was particularly drawn to the performance of the Canadian equestrian jumper Eric Lamaze, former world number one, who was attempting a comeback after his Olympic gold medal horse Hickstead died suddenly in November of 2011.

My aim was to obtain enough data to describe the kinematics and dynamics of jumping over a high fence and a wide water barrier using basic physics and elementary mathematics suitable for high school physics students. However, no kinematic or dynamic information for the jumping at this event was available. I considered taking a high-speed camera and a Doppler shift apparatus to Calgary this spring to measure speeds. But soon I discovered that acquiring the necessary apparatus as well as the permission to be granted access to a thoroughbred horse and a rider was not a realistic proposition.

Luckily we recorded some of the equestrian competitions on our TV external drive. I soon realized that I could study the motion of Eric Lamaze and his new young mare, Derly, in a Grand Prix event in which he and his new horse placed second. This event, the CN Grand Prix of June 12, 2012, can be found on YouTube and the reader is encouraged to study the motion of Eric and his horse.

Furthermore, two articles were very helpful in providing additional data to exploit in my study. These data were for jumping heights of 1.40 m. This together with the results I obtained has enabled me to provide explanations suitable for introductory physics students for both the kinematics and dynamics of jumping based on the principles of Newtonian mechanics.

Data acquisition

I discovered that the pause button on the remote control of my external drive enabled me to measure motion at a time interval of about 1/50 second.

For the kinematics and dynamics of the clearing of the fence, we need to know:

1. ***The height of the fence.***
   The height was 1.60 m, the maximum height permitted for the event.
2. ***The speed of the horse just before liftoff.***
   An investigation of the kinematics of horse jumping[1] reports an average approach speed of 3.7 m/s. I assumed a speed of 4.0 m/s for our case.
3. ***The angle of elevation of the horse just before takeoff.***
   The average angle of elevation for the three cases in Ref. 1 was about 40-45°.[1] For our case this angle was easily found by measurement to be about 40°. In Fig. 2 the angle of Derly's body is indeed about 40°.
4. ***The time of contact between the hind hooves that is needed for push-off.***
   This time was estimated by counting the number of pauses required for the hind hooves to produce liftoff, and was found to be about 0.20 s. This measurement is crucial for determining the forces involved.
5. ***The time of flight of the horse and rider.***
   Again, using the pause button, I found that the time it took for the horse to jump the 1.60-m fence was about 0.70 s. This value will also be confirmed by the kinematic calculations.
6. ***The total distance from the contact point of takeoff and the landing on the front hooves.***
   This distance was estimated to be about 4.8 m. This will later be confirmed by kinematic calculations. In addition, I needed to know the mass and the height of the horse, as well as the mass of the rider. Derly's mass is about 500 kg and her height is given as 17 hands (1.73 m). With the mass of Eric and the saddle (70 kg), the combined mass of the horse and rider system is about 570 kg.

Description of equestrian jumping

Equestrian jumping involves four phases described in a comprehensive article by Dr. Sheila Schills, a well-known expert in equine rehabilitation.[3] They are the approach, the takeoff, the flight, and the landing. They are illustrated in

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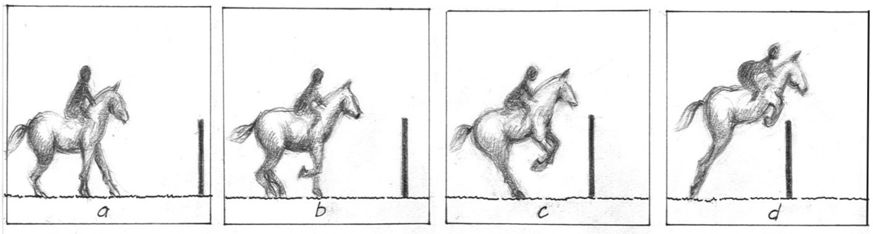

Fig. 1. Part 1. The approach and the takeoff.

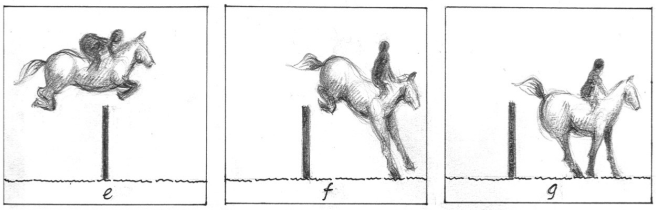

Fig. 1. Part 2. The flight and the landing.

Fig. 1 as a sequence of seven sketches labeled (a) through (g). Approach is illustrated in (a) and (b), takeoff in (c) and (d), flight in (e), and landing in (f) and (g). They can be summarized as follows:

Approach

The horse must get to the fence at an even and steady gait (usually a canter motion) so that she can concentrate on the best spot to take off for the jump. (See Fig. 1.) The horse reaches forward and down with her neck to lower the front legs and her CM (center of mass).The front legs are propped or strutted out in front of the body. This relatively sudden braking action allows momentum to carry the hind legs further under the body of the horse than would be otherwise possible.

Takeoff

As the horse finishes the last complete stride before the jump, she will begin to shift the weight backward by raising the head, shortening the neck, and lifting the shoulders. The horse's neck continues to shorten to assist in moving the weight backward. This shortening of the neck also helps to abort the normal forward movement of the canter. As the weight moves backward, the hind legs compress or coil. With the maximum amount of flexion in the hind joints, the horse can then create the maximum push against the ground to propel her up and forward over the jump. The horse can have the most effective takeoff when the hip joint is placed vertically above the hoof.

Fig. 2. Eric Lamaze and Derly jumping a 1.60-m fence. The takeoff angle is almost exactly 40º from the horizontal and the distance from the fence to the hooves about 2.4 m. The center of mass of the pair is indicated by the blue dot on the horse. (Photo courtesy of Franz Venhaus)

Flight

The hind legs reach maximum extension after they leave the ground and the front legs are curled tightest against the body. The knees lift and bend to curl the legs up, the tighter the better, so the chance of hitting the fence by the front legs is reduced. To bend the knees and lift the forehand, the scapula (shoulder) rotates upward and forward. During the flight the CM follows an approximate parabolic trajectory.

Landing

To slow the forward momentum so that the force of impact is reduced, the horse will swing the neck and head up as the forelegs reach toward the ground. The non-leading front leg lands first. When the leading front leg lands, both legs push against the ground in a downward and backward direction. The hindquarters rotate underneath the trunk and reach toward the ground as the forehand moves forward and out of the way of the hindquarters.

Analysis of jumping

Equestrian jumping involves clearing hurdles (and water barriers) as the beautiful photograph shown in Fig. 2 of Derly and Eric Lamaze illustrate. It can be likened to hurdle jumping and the long jump of human athletics. Accordingly, they are analyzed sequentially: first the high fence followed by the water barrier jump.

High fence jump

The particular jump we choose for analysis here is one of the 1.60-m fences that was used in the Grand Prix event at Spruce Meadows. Derly approached this high fence with a speed of about 6.0 m/s. The speed was reduced by a shorter stride to about 4.0 m/s just before anchoring the hind legs in position. We assume that the horizontal speed of 4.0 m/s does not change significantly during the 0.20-s push-off period.

The front legs were lifted and the hind legs stopped moving for about 0.20 s during what is called a "stance phase." The front legs are coiled so that the body of the horse with reference to the horizontal can be as high as 45º, just before push-off. The hind legs uncoil, and at the point of leaving the ground, the angle for the trajectory now is about 40º. During this stance phase, the CM of the horse-rider system moves about 0.80 m (4.0×0.20).

The CM of Derly at the moment of takeoff is estimated to be about 20 cm along the line of Derly's body, in front of Eric's

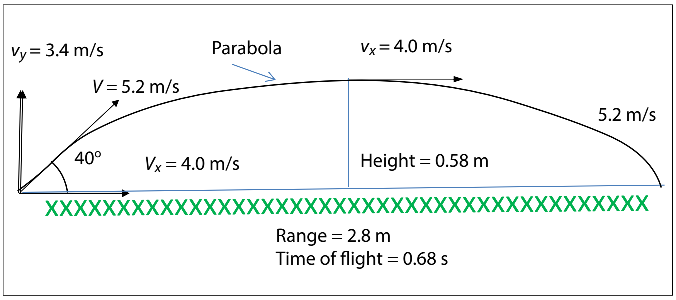

Fig. 3. The motion of the CM of horse and rider over the 1.60-m fence.

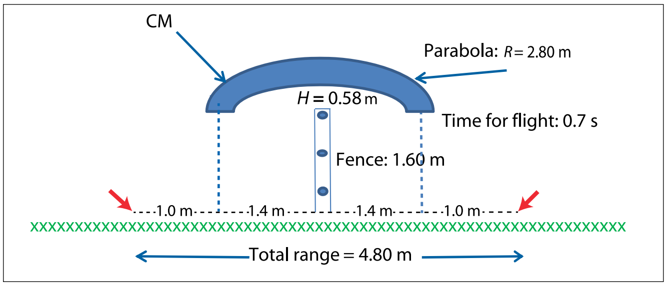

Fig. 4. The kinematics of jumping a 1.60-m fence is represented in this example of an ideal jump. Red arrows indicate launching point and landing point of hooves and the CM motion is given by the blue arc.

right foot. This was done by scaling, that is by comparing distances to the height of the 1.60 bar, using the photo of Eric Lamaze and Derly shown in Fig. 2. Knowing the angle at takeoff and the horizontal velocity, we can easily determine the instantaneous vertical velocity at the beginning, the height, the range, and time of flight for the trajectory.

The kinematics of the jump

Derly pushes off when the angle to the horizontal is about 40° and the horse-rider system's CM at this moment happens to be at about the same height as that of the fence. The horizontal velocity is constant at about 4.0 m/s. Where relevant in calculations, it is appropriate (given the 5% accuracy we are working with) to take g, the acceleration of gravity, to be 10 m/s^2. The vertical velocity at the moment of liftoff is related to the horizontal speed component

$$v_y = v_x \tan 40^\circ. \tag{1}$$

Therefore v_y = 4.0 tan 40° = 3.4 m/s and the time t to reach height h is obtained from

$$v_y = gt \tag{2}$$

or

$$t = v_y / g = 3.4/10 = 0.34 \text{ s}.$$

The total time for the trajectory is $2t$, i.e., ~0.7 s as expected. Applying the equation of uniformly accelerated motion in the vertical direction, we have

$$v_y^2 = 2\,gh, \tag{3}$$

so the height reached by CM is

$$h = (3.4)^2 / 20 = 0.58 \text{ m}.$$

The total height from the ground to the center of gravity, assuming the initial CM is approximately the same as the fence height, is 1.60 + 0.58 = 2.18 m.

The horizontal range of the CM is given by

$$R_{CM} = 2 v_x t. \tag{4}$$

Thus $R_{CM} = 4.0 \times 0.7 = 2.8$ m. The tangential velocity v at the point of liftoff is

$$v = (v_x{}^2 + v_y{}^2)^{1/2}. \tag{5}$$

Thus $v = (4.0^2 + 3.4^2)^{1/2} = 5.2$ m/s.

Remember that the horizontal distance from the spot where the hind leg hooves take off to the CM, that is, the start of the trajectory, is about 1 m. If the jump is perfectly symmetrical (and it seldom is), the total range R is given by: R = 2.8 + 2.0 = 4.8 m.

It should be noted that the location of the CM of Derly does change somewhat during the flight because the horse's body configuration changes due to the movement of the neck and the leg during the flight. Therefore, the trajectory is only an approximate parabola, as indicated in Fig. 4.

The dynamics of the jump

The motion during the push-off stage that takes about 0.20 s is fairly complicated. However, the sketches of Fig. 1 suggest that the direction of the force produced by the hind legs varies by only a small angle (say 15°) about the vertical. However, its line of action is behind the CM by a distance d, which advances significantly during this short time of contact. At the start of the push, Derly's body is at an elevation angle of about 45° [see Figs. 1(c) and 1(d)]. As the hind legs are pushing forward (remember the hooves are stationary during this brief period), the body moves about 0.80 m (4.0 m/s$\times$0.20 s) forward. The product $d \times F$, variable in itself during the push-off duration of 0.20 s, is a torque that causes Derly to rotate clockwise after the initial counterclockwise motion produced by the forelegs push-off prior to the jump. When the hooves leave the ground, the CM has moved about 0.80 m horizontally and Derly is moving with an initial vertical velocity component of 3.4 m/s and a constant horizontal velocity component of 4.0 m/s, essentially along a parabolic trajectory. When contact with the ground has ended, the direction of the body of the horse is about 40° with the horizontal.

We assume that for the liftoff most of the force acts in the vertical direction and ignore the horizontal force. The average thrust force during the 0.20-s push-off period can be obtained by using the relationship between impulse and change of momentum:

$$F\,\Delta t = m\,\Delta v. \tag{6}$$

Therefore

$$F = m\,\Delta v/\Delta t. \tag{7}$$

The total vertical force F_y acting during the impulsive action to propel the CM of the horse to the elevation h is:

$$F_y = mg + m\,\Delta v/\Delta t \tag{8}$$

$$\Delta v/\Delta t\ = 3.4/0.20 = 17\ \text{m/s}^2.$$

Therefore $F_y = m(g + \Delta v/\Delta t) = 570(10 + 17) = 15{,}400\ \text{N} = 15.4\ \text{kN}$.

This is a large average force that acts during the 0.20-s contact. The force varies during this short time of 0.20 s and reaches a peak of perhaps 19 kN, at about 0.10 s. Therefore, each leg must be able to support a force of about 7700 N in a symmetric case. The total energy supplied by the hind legs for the jump is given by the maximum gravitational energy the CM of the horse and rider assumed during the flight:

$$E = mgh. \tag{9}$$

Therefore the energy supplied by the jump is $E = 570\times 10\times 0.58 = 3300$ J.

The average force on landing that acts on the front legs, however, is a little larger, because the horse's horizontal velocity component typically slows down to about 3 m/s upon landing during the 0.20-s contact. The vertical force F_y will be, as before, about 15,400 N, but we also have a horizontal force acting because of the reduction of the velocity by about 1 m/s. The average horizontal force is

$$F_x = m\,\Delta v/\Delta t = 570\times 1/0.20 = 2900\ \text{N}.$$

The total force is

$$F = (F_x{}^2 + F_y{}^2\,)^{1/2} \tag{9}$$

or

$$F = (2900^2 + 15{,}400^2)^{1/2} = 16{,}000\ \text{N}.$$

Is this large force reasonable? The force measurements done for jumping over a 1.40-m fence, as reported in Ref. 2, are given as an average of 14 kN. So they are consistent with our higher value of 16 kN given the higher fence height.

The ideal liftoff distance

The kinematics of elementary trajectory motion requires that the CM of the horse in our case clear the fence by a height of 0.58 m in about 0.7 s. If we want to maintain this height,

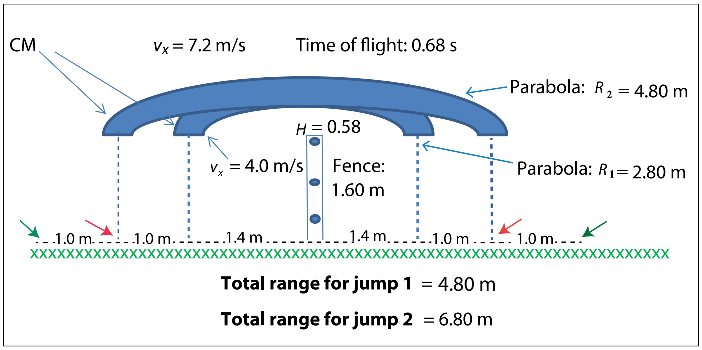

Fig. 5. Comparing the kinematics of two jumps for the same height above the fence. Red arrows are the launching and landing points for the small parabola, while the green arrows represent the launching and landing points for the large parabola.

the time of the trajectory does not change with the push-off distance (see Fig. 5). This is a well-known discrepant event and always astonishes students when they see a demonstration, using balls rolling off a table at different speeds.

It is useful to see how small changes to this description affect the scenario. For example, if the distance of the CM on takeoff is only 0.50 m closer to the fence, then the angle at takeoff would be over 50°. On the other hand, if the horse jumps from a distance 1.0 m behind the optimum distance of about 2.4 m, the angle would be about 30°, but the horizontal velocity would have to be 7.2 m/s. Since the horse generally slows down by about 2 m/s just before the takeoff stance, the approach velocity would have to be at least 9 m/s. This velocity usually requires galloping and results in lessening the ability of the horse to assume a symmetric stance for takeoff.

In addition, the forces acting on the front legs would be a little larger, because the horse typically reduces the landing speed to about 3 m/s. That means that there is a greater horizontal force than in the optimal case: $F_x = m\,\Delta v/\Delta t = 570(7.2 - 3)/0.20 = 12{,}000\ \text{N}$. The vertical force is, as for the hind legs, 15,400 N.

The total force acting on the front legs then would be:

$$F = F = (F_x{}^2 + F_y{}^2\,)^{1/2} = (12{,}000^2 + 15{,}400^2\,)^{1/2} = 20{,}000\ \text{N}.$$

This is a considerably larger force acting on the front legs than when jumping the shorter trajectory. Therefore, if Eric chooses the longer jump to gain advantage in time, he risks his horse having to encounter greater retarding forces, especially on landing.

Finally, it should be mentioned that just as in the case of the takeoff force acting behind the CM of the horse, producing a clockwise rotation, the contact force produced by the front legs on landing acts in front of the CM and produces a counterclockwise rotation. Indeed, if the angle of descent is large enough, the horse will rotate clockwise, which results in a dangerous summersault, with the potential of severe injury to both rider and horse.

Water jumping

If equestrian jumping is like jumping hurdles, then water jumping is very similar to the long jump. The width of the jump at the Grand Prix at Spruce Meadows was 4.2 m. So Eric and Derly had to make sure that the jump was at least 5.0 m long (Fig. 6 shows a jump of 6.2 m). The angle of elevation at takeoff was about 25° and the approach speed about 7.5 m/s, because for a range of the trajectory to be 5.0 m, at an angle of 25°, the horizontal velocity must be 7.5 m/s . Following the same reasoning as before, using Eqs. (1)–(3), we get that the time of flight is $t = 0.7$ s, $v_y = 3.5$ m/s initially, and the height of the trajectory $h = 0.61$ m:

$5.0 = v_x t = 7.5\,t$ Therefore $t = 0.7$ s.
Since $\tan 25° = v_y / v_x$, $v_y = 3.5$ m/s
and the height h of the trajectory is $h = v_y^2 / 20 = 0.61$ m.

The contact time for the takeoff is also about 0.20 s and therefore the vertical force necessary for the trajectory is given by

$$F_y = mg + m\,(3.5/0.20) = 570(10 + 17.5) = 15{,}600 \text{ N},$$

very much the same as for the 1.60-m fence.

On landing, the vertical force F_y, as for the hind legs on takeoff, is about 15,600 N. As before, the horse is reducing her speed, this time to about 5.0 m/s, from 7.5 m/s. Therefore, the horizontal force is

$$F_x = 570(2.5/0.20) = 7100 \text{ N}.$$

The total force then is $F = (15{,}600^2 + 7100^2)^{1/2} = 17{,}000$ N, a little larger than the force required for the fence jumping.

These large forces acting on the horses, even if only for a very short time, are stressful for them. Riders are very concerned about their horses and make sure that they are healthy, both physically and emotionally. Horses are examined by veterinarians before each competition. Serious accidents in Grand Prix jumping, unlike in steeple chasing and in racing, do occur but are rare.

Conclusion

While the data used and the estimates made by the author for these calculations are inadequate for a technical journal in biomechanics, the author hopes that after studying the physics of equestrian jumping in this way, the enjoyment of students watching a Grand Prix will be enhanced.

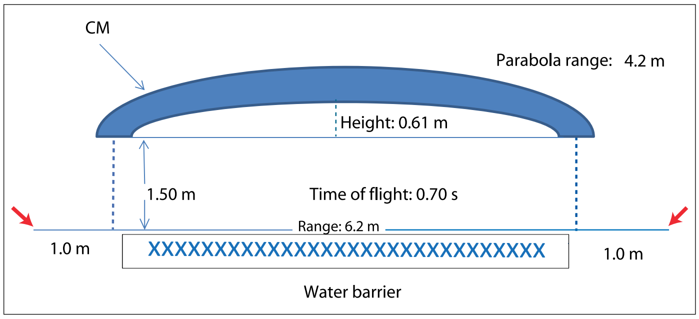

Fig. 6. The kinematics of an ideal jump across a 4.2-m water barrier with launching and landing points shown by red arrows.

Note: Since the writing of this article, Eric Lamaze has sold his Grand Prix horse, Derly, and has acquired a number of young and potentially top Grand Prix jumpers. He is now competing in Florida in preparation for the summer season in his favorite venue, Spruce Meadows in Alberta.

Acknowledgments

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