Quantifying Equestrian Show Jumping: A Large Context Problem for Physics Students

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Equestrian show jumping has become a popular spectator sport in Canada since the Beijing Olympic Games in 2008. Canada received a gold medal in singles and a silver in team competition. Eric Lamaze and his stallion Hickstead, generally regarded as the best show-jumping horse of his generation, became internationally famous, and Lamaze was ranked first in the world. Unfortunately, three years later Hickstead suddenly died in Verona, Italy, after jumping a clear round. This tragic event plunged the equestrian community into deep mourning.

These events reawakened the love for horses I acquired when working in the forestry industry in British Columbia as a young man. As a physics educator, I naturally became interested in the physics of the jumping motion of these magnificent animals.

I remember a letter written by an irate reader of the British journal *New Scientist* in response to my article “Physics and the Bionic Man” (Stinner 1980). The gentleman argued that my testing the feats claimed by the bionic man, using the laws of physics, spoiled the enjoyment of many devotees of the popular TV series *The Six Million Dollar Man*.

As students of physics, we can always appreciate the aesthetics of phenomena such as rainbows and sunsets, but understanding the physics should enrich our aesthetic appreciation. Similarly, equestrian show jumping can be appreciated on more than one level.

The Background Story

Last September I went to see some jumping events at Spruce Meadows, near Calgary, which is considered the Wimbledon of show jumping. I was especially interested in understanding the kinematics and dynamics of the jumping. I wanted to see if, as in the case of the physics of the bionic man, it was possible to establish a context, embedded in a good storyline that would interest many students.

The central idea behind contextual teaching of physics is that a context that attracts students’ interest and sparks the imagination can be developed in such a way that questions and problems arise from the context naturally, not in a contrived way (as in textbooks). Also, the problems generated have no obvious answer (even to the instructor) and can be solved using basic physics and mathematics. The reader is encouraged to visit my website to see the many large context problems I have developed over the years.[1]

It was easy to get data for the study of the dynamics of the bionic man by simply watching the TV series. It was also easy to show that the feats claimed for him were physically impossible. However, the feats of the show-jumping horses were there for all to see.

Nevertheless, the data for the jumping were not available. I considered taking a high-speed camera and a Doppler shift apparatus to Calgary to measure speeds—but I soon discovered that because of the apparatus required, as well as requiring access to a thoroughbred horse and a rider, this was not a realistic proposition.

My aim was to obtain enough data to describe the kinematics and dynamics of a horse’s jumping over a high fence and a wide water barrier, using basic physics and elementary mathematics suitable for high school physics students. I managed to combine the data from articles on biomechanics with simple observation of jumping on a CBC TV presentation of a Grand Prix event at Spruce Meadows. Using the TV remote control and the pause button, which responded to a time interval of 1/50 s, I was able to estimate the speed, the time of flight and the time taken for a horse's hind legs to "stop" moving just before the jump could be estimated. The results I obtained were reasonably good for both the kinematics and the dynamics of jumping.

Luckily, I had recorded other equestrian competitions. I studied the motion of Eric Lamaze and his new young mare, Derly, in a Grand Prix event in which they placed second. That event—the CN Grand Prix of June 12, 2012—can be found on YouTube. I encourage readers to watch and study Lamaze and his horse.

Moghaddam and Khosravi (2007) were useful in estimating takeoff velocity, height of the centre of gravity (CG) trajectory, time of flight and range of the jump. Meershoek et al (2001) provided data for the forces on the horse's legs on landing. These data were for jumping heights of 1.4 m.

For the kinematics and dynamics of clearing a fence, we need to know the following:

- *The height of the fence.* The height of the fence was 1.6 m, the maximum height permitted for the event.
- *The speed of the horse just before liftoff.* An investigation of the kinematics of horse jumping reports an average approach speed of 3.7 m/s (Moghaddam and Khosravi 2007). I assumed a speed of 4.0 m/s for our case.
- *The angle of elevation of the horse just before takeoff.* The average angle of elevation in Moghaddam and Khosravi (2007) was 40–45°. For our case, this angle was easily found by measurement to be about 40°. In Figure 1, the angle of Derly's body is indeed about 40°.
- *The time of contact between the hind hoofs that is needed for push-off.* This time was estimated by counting the number of pauses required for the hind hoofs to produce liftoff, and was found to be about 0.2 s. This measurement is crucial for determining the forces involved.
- *The time of flight of the horse and rider.* Moghaddam and Khosravi (2007) report an average time of flight of about 0.8 s. Using the pause button, I found that the time it took for the horse to jump the 1.6 m fence was about 0.7 s. This value will be confirmed by kinematic calculations.
- *The total distance from the contact point of takeoff and the landing on the front hoofs.* This distance could be estimated to be about 5.0 m—again to be confirmed by kinematic calculations.

I also needed to know the weight (mass) and height of the horse, as well as the weight of the rider. Derly weighs about 500 kg, and her height is given as 17 hands (1.73 m). With the mass of Lamaze and the saddle, the combined mass is about 570 kg.

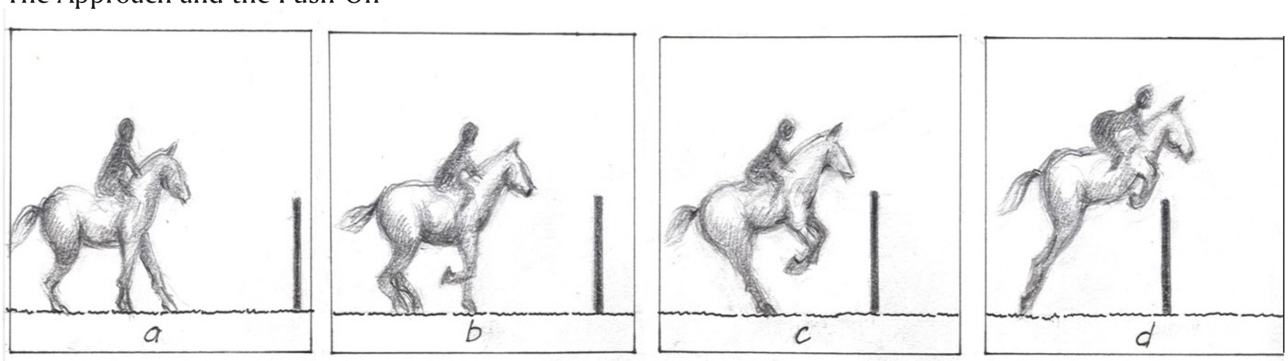
Figure 1

Eric Lamaze and Derly jumping a 1.6 m fence. The takeoff angle is almost exactly 40° from the horizontal, and the distance from the fence to the hoofs is about 2.5 m. The CG of the pair is roughly 20.0 cm along Derly's body, where Eric's right foot is. Photo courtesy of Franz Venhaus.

Jumping Fences

The following descriptions of the approach, takeoff, flight and landing during a jump are adapted from a comprehensive piece by Sheila Schils, a well-known expert in equine rehabilitation, entitled "Biomechanics of Jumping."[2]

The Approach and the Push-Off



The Flight and the Landing

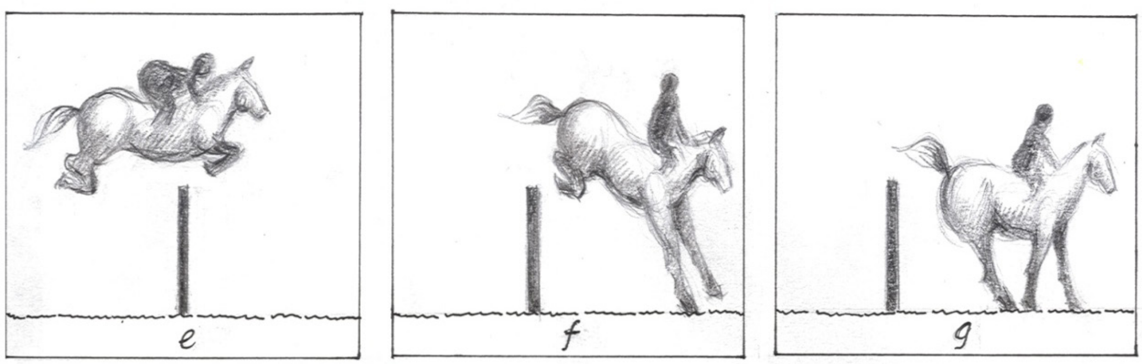


Figure 2

Fence jumping: approach, push-off, flight and landing

Approach

The horse must reach the fence at an even, steady gait (usually a canter motion) so that she can focus on the best spot for takeoff. See Figure 2a.

The horse reaches forward and down with her neck in order to lower the front legs and her CG. The front legs are propped or strutted out in front of the body. This relatively sudden braking action allows momentum to carry the hind legs further under the body of the horse than would otherwise be possible.

Takeoff

See Figures 2b, 2c and 2d.

As she finishes the last whole stride before the jump, the horse begins to shift her weight backward by raising her head, shortening her neck and lifting her shoulders.

Her neck continues to shorten to help move the weight backward. This also helps to stop the normal forward movement of the canter.

As the weight moves backward, the hind legs compress or coil. With the maximum amount of flexion in the hind joints, the horse can then create the maximum push against the ground to propel herself up and forward. The horse has the most effective takeoff when her hip joint is placed vertically above the hoof.

Flight

See Figures 2d, 2e and 2f.

The horse's hind legs reach maximum extension after they leave the ground, and her front legs are curled tight against her body.

Her knees lift and bend to curl the legs up, the tighter the better, to reduce the chance of her hitting the fence with her front legs. To bend her knees and lift her forehand (the front part of the horse's body), the scapula (shoulder) rotates upward and forward. During the flight, the CG follows an approximate parabolic trajectory.

Landing

See Figures 2f and 2g.

To slow the forward momentum and reduce the force of impact, the horse swings her neck and head up as her forelegs reach toward the ground.

The horse's nonleading front leg lands first. When her leading front leg lands, both legs push against the ground in an upward and backward direction. The hindquarters rotate underneath the trunk and reach toward the ground as the forehand moves forward and out of the way of the hindquarters.

Jumping Fences: Physical Principles

Approach and Takeoff

I am referring, for analysis, to one of the 1.6 m fences used in the Grand Prix. Derly approached this high fence with a speed of about 6.0 m/s. Her speed was reduced by a shorter stride to about 4.0 m/s just before she anchored her hind legs. We assume that the horizontal speed of 4.0 m/s does not change significantly during the 0.2 s push-off.

In Figure 1, Derly's front legs are lifted and her hind legs stop moving for about 0.2 s. This is called the stance phase. The front legs are coiled so that the body of the horse, with reference to the horizontal, can be as high as 45°, just before push-off. The hind legs uncoil, and at the point of leaving the ground, the angle for the trajectory is about 40°. During this stance phase, the body moves about 0.8 m (4.0 m/s × 0.2 s).

At the moment of takeoff, Derly's CG is about 20.0 cm along the line of her body, in front of Lamaze's right foot.

Knowing the angle at takeoff and the horizontal velocity, we can easily determine the instantaneous vertical velocity at the beginning, the height, the range and the time of flight for the trajectory.

The Kinematics of the Jump

See Figure 3.

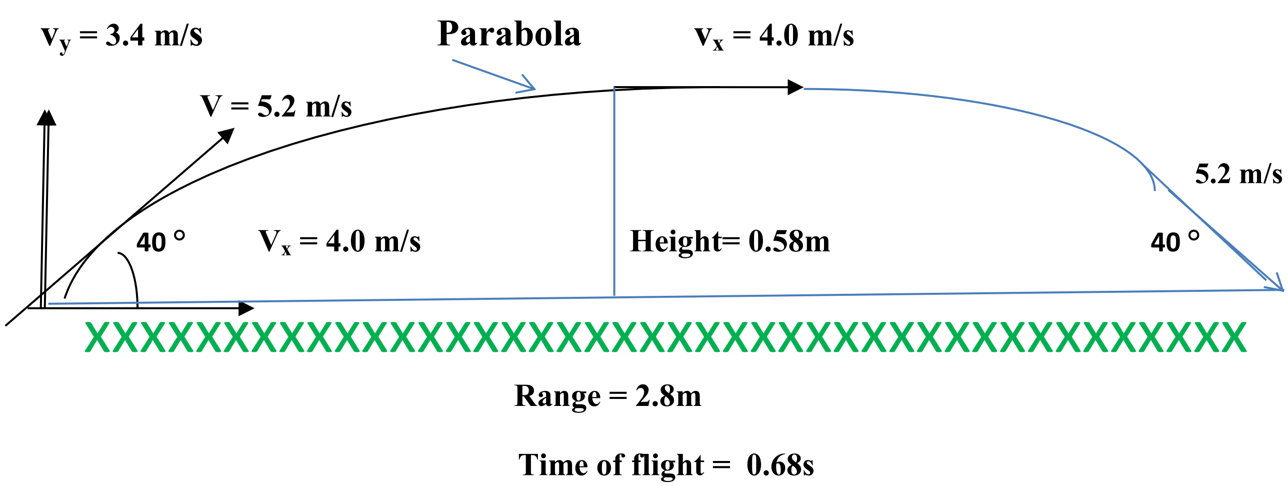


Figure 3

The motion of the CG of horse and rider over a 1.6 m fence

Derly pushes off when the angle to the horizontal is about 40°, and the CG at this moment happens to be at about the same height as the fence. The horizontal velocity (v_x) is constant at about 4.0 m/s. (We will assume that the value of g, the gravitational attraction, is $g = 10.0$ m/s^2.)

The vertical velocity (v_y) at the moment of liftoff is given by

$$v_y = v_x \tan 40°. \quad (1)$$

Therefore,

$$v_y = 4.0 \tan 40° = 3.4 \text{ m/s}$$

and the time t to reach height h is obtained from

$$v_y = gt \quad (2)$$

or

$$t = v_y/g = 3.4/10.0 = 0.34 \text{ s}.$$

The total time for the trajectory is $2t$, or 0.68 s.

Since

$$v_y^2 = 2gh, \quad (3)$$

the height reached by the CG is

$$h = 3.4^2/20.0 = 0.58 \text{ m}.$$

The total height from the ground to the CG is

$$1.60 + 0.58 = 2.18 \text{ m}.$$

The range R of the CG is given by

$$R_{CG} = 2v_x t \quad (4)$$

or

$$R = 4.0 \times 0.68 = 2.72 \text{ m}.$$

The tangential velocity v_T at the point of liftoff is

$$v_T = (v_x^2 + v_y^2)^{1/2}$$

or

$$v_T = (4.0^2 + 3.4^2)^{1/2} = 5.2 \text{ m/s}.$$

Remember that the distance from the spot where the hind leg hoofs take off and the horizontal distance to the CG (that is, the start of the trajectory) is about 1.0 m. If the jump is perfectly symmetrical (and it seldom is), the distance from the base of the fence to the point of contact, in this case, is about 2.72 m. The total range R is

$$R = 2.72 + 2.0 = 4.8 \text{ m}.$$

It should be noted that the location of Derly's CG (see Figure 1) does change somewhat during the flight as the horse's body configuration changes (due to the movement of the neck and the leg during the flight). Therefore, the trajectory is only an approximate parabola, as indicated in Figure 4.

The Dynamics of the Jump

The motion during the push-off stage that takes about 0.2 s is fairly complicated. Looking at Figure 2, it is clear that the force F produced by the hind legs is almost vertical. However, it misses the CG by a small distance d, which increases during this short time of contact. At the start of the push, Derly's body is along an elevation of about 45° (see Figure 5). As the hind legs push forward (remember that the hoofs are

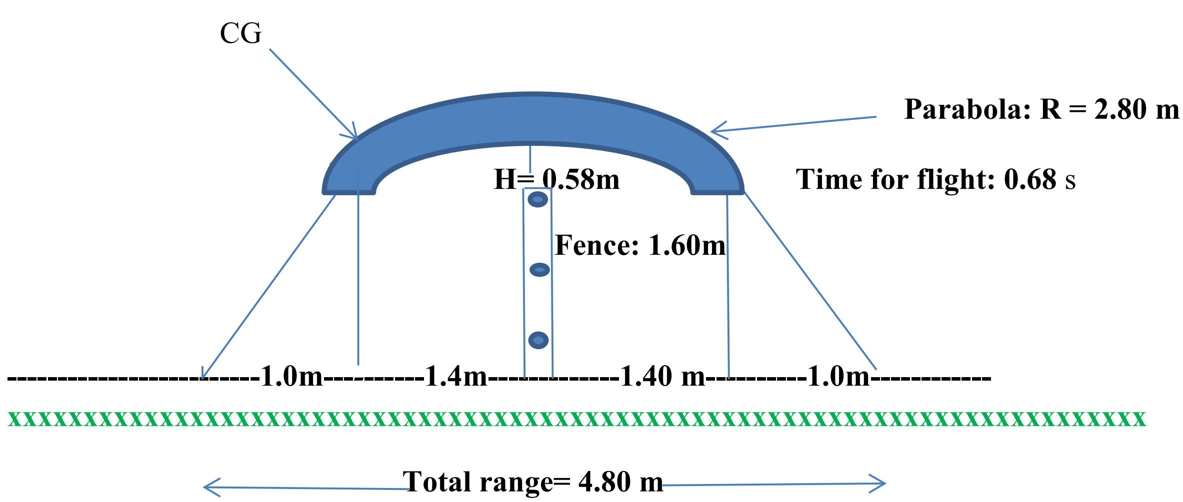


Figure 4

The kinematics of jumping a 1.6 m fence

stationary during this brief period), the body moves about 0.8 m forward. The product of *dF* is a torque, which causes Derly to rotate clockwise for about 0.2 s. When the hoofs leave the ground, the CG has moved about 0.8 m horizontally and Derly is moving with an initial vertical velocity of 3.4 m/s and a constant horizontal velocity of 4.0 m/s—essentially along a parabolic trajectory. The resultant initial velocity, then, must be 5.2 m/s, as shown earlier.

When contact with the ground has ended, the direction of the body of the horse is about 40° with the horizontal.

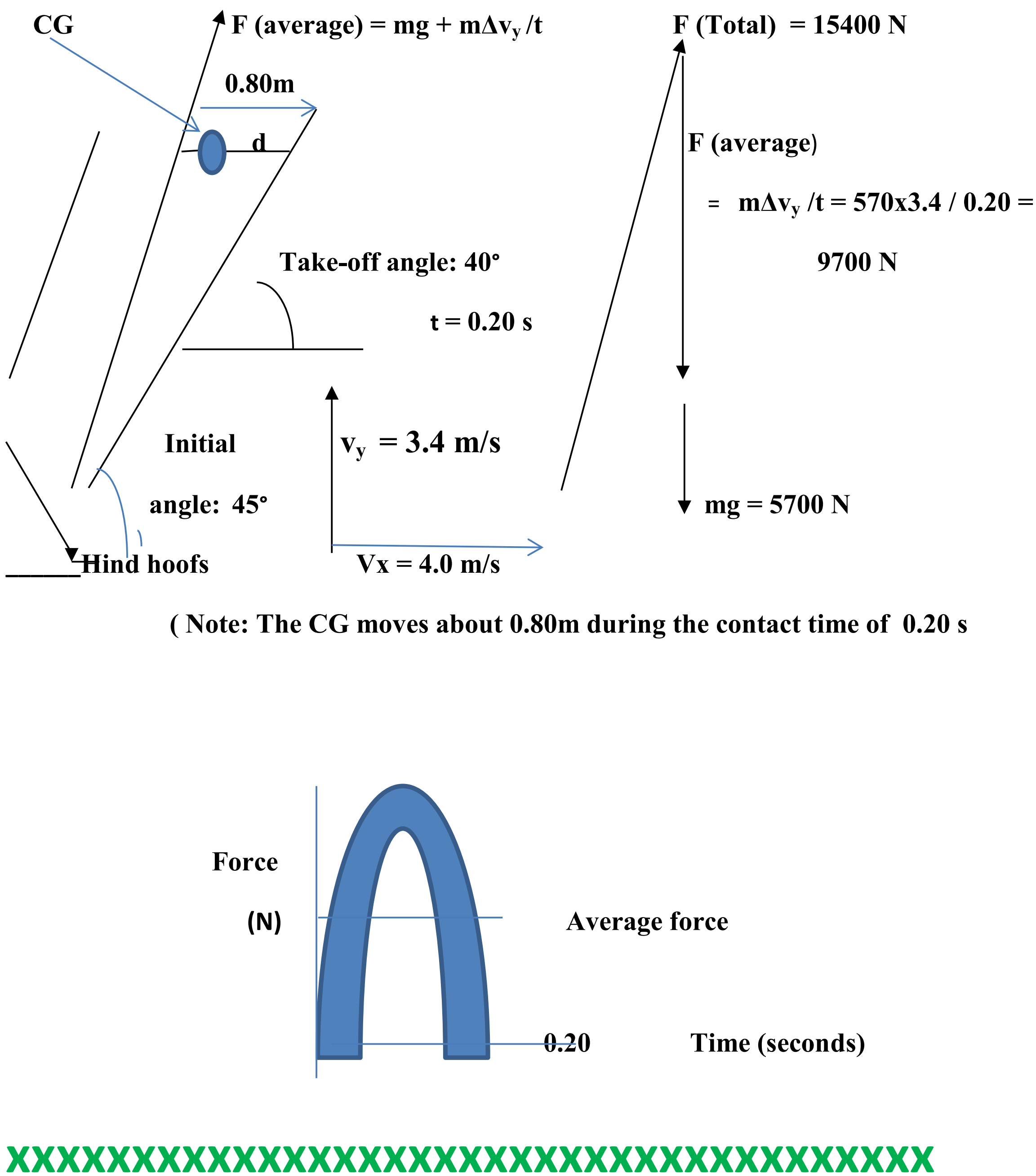


Figure 5

Forces acting during takeoff (0.2 s)

The impulse, $F\Delta t$, is equal to the rate of change of momentum, or $F\Delta t = m\Delta v$, where F is an average force.

We can estimate the average force acting on the legs to produce the liftoff by finding the vector sum of the vertical and horizontal force during contact. In addition, we assume that, for the liftoff, most of the force acts in the vertical direction and we can ignore the horizontal force. The combined mass of Derly, Lamaze and the saddle is about 570 kg.

The average push force during the 0.2 s can be obtained by using the relationship between impulse and change of momentum:

$$F\Delta t = m\Delta v.$$

Therefore,

$$F = m\Delta v/\Delta t.$$

The vertical force necessary to propel the CG of the horse to the height h is

$$F_y = mg + m\Delta v/\Delta t$$
$$\Delta v/\Delta t = 3.4/0.2 = 17.0.$$

Therefore,

$$F_y = m(g + \Delta v/\Delta t) = 570(10.0 + 17.0) = 15{,}400 \text{ N}.$$

This is a large average force that acts during the 0.2 s contact. The force varies during this short time and peaks at perhaps 19,000 N, at about $t = 0.1$ s, as shown in Figure 5. Therefore, the force calculated is an average force.

The force is equivalent to about 15,000 N, or a 1,500 kg-force, or about 3,300 lb. Therefore, each leg must be able to support a force of about 750 kg-force in a symmetric case.

The total energy expended by the hind legs for our jump is given by

$$E = mgh.$$

Therefore, the energy produced for the jump is

$$E = 570 \times 10 \times 0.58 = 3{,}306 \text{ J}.$$

It is interesting to calculate the average power generated during this jump. Since about 3,300 J of energy is produced by the push-off in 0.2 s, the power is 3,300/0.2 = 16,500 W, or about 22 HP.

About 7.0 W/kg is produced by each hind leg when jumping a fence 1.4 m high.

The average force on landing that acts on the front legs, however, is a little larger, because the horse typically slows down to about 3.0 m/s during the 0.2 s contact. See Figure 6.

The vertical force F_y is, as before, about 15,400 N, but we also have a horizontal force acting because of the reduction of the velocity by about 1.0 m/s. The average horizontal force is

$$F_x = m\Delta v/\Delta t = 570 \times 1.0/0.2 = 2{,}900 \text{ N}.$$

The total force is

$$F = (F_x^{\,2} + F_y^{\,2})^{1/2}$$

or

$$F = (2{,}900^2 + 15{,}400^2)^{1/2} = 16{,}000 \text{ N}.$$

Is this large force reasonable? The force measurements for jumping over a 1.4 m fence, as reported in Meershoek et al (2001), are given as an average of 14 kN.

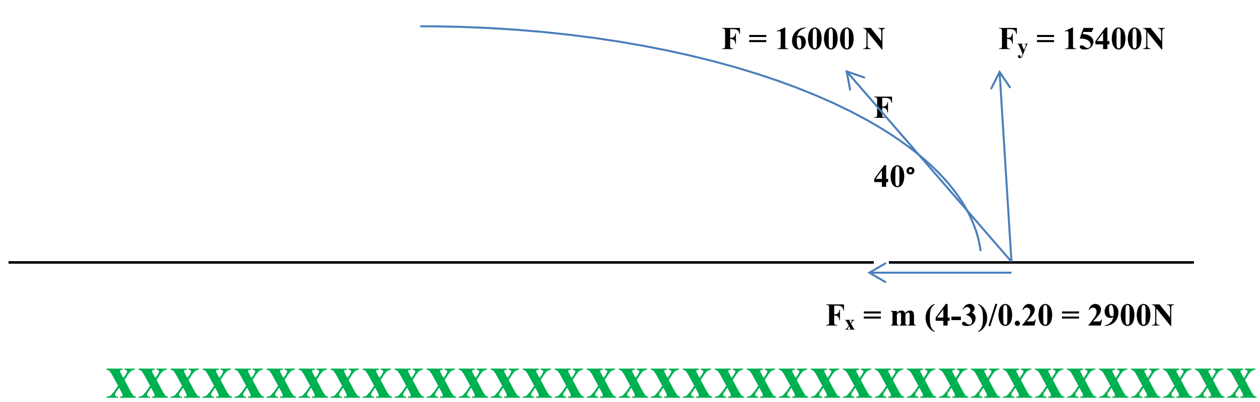


Figure 6
Forces acting on landing

The Ideal Liftoff Distance

The kinematics of elementary trajectory motion require that the CG of the horse in our case clear the fence by a height of 0.58 m in 0.68 s. If we want to maintain this height, the time of the trajectory does not change with the push-off distance (see Figure 7). This is a well-known discrepant event, and it always astonishes students when they see a demonstration, using balls rolling off a table at different speeds.

We can assume that the ideal liftoff distance from this fence is about 1.4 m (that is, the distance from the CG to the fence at the moment of liftoff). The hind hoofs must be anchored at a distance of about 2.5 m from the fence. The angle of elevation of the horse at the beginning of the push will be about 45°, and at the moment of liftoff it will decrease to about 40°. The clockwise rotation of the horse's body during this brief 0.15 s contact time is due to the torque produced by the upward push of the legs, the direction of this force missing the CG by a small amount.

For example, if the distance of the CG on takeoff is only 0.5 m closer to the fence, then the angle at takeoff will be over 50°. On the other hand, if the horse jumps from a distance 1.0 m behind the optimum distance of about 2.4 m, the angle will be about 30°, but the horizontal velocity will have to be 7.2 m/s. Since the horse generally slows down by about 2.0 m/s just before the takeoff stance, the approach velocity would have to be at least 9.0 m/s. This velocity usually requires galloping and results in lessening the horse's ability to assume a symmetric stance for takeoff.

In addition, the forces acting on the front legs will be a little larger, because the horse typically reduces the landing speed to about 3.0 m/s. That means that there is a greater horizontal force than in the optimal case:

$$F_x = m\Delta v/\Delta t = 570(7.2 - 3.0)/0.20 = 12{,}000 \text{ N}.$$

The vertical force, as for the hind legs, is 15,400 N.

The total force acting on the front legs then is

$$F = (F_x^2 + F_y^2)^{1/2} = (12{,}000^2 + 15{,}400^2)^{1/2} = 20{,}000 \text{ N}.$$

This is a considerably larger force acting on the front legs than when jumping the shorter trajectory.

Therefore, if Lamaze chooses the longer jump to gain advantage in time, he risks his horse having to encounter greater retarding forces, especially on landing.

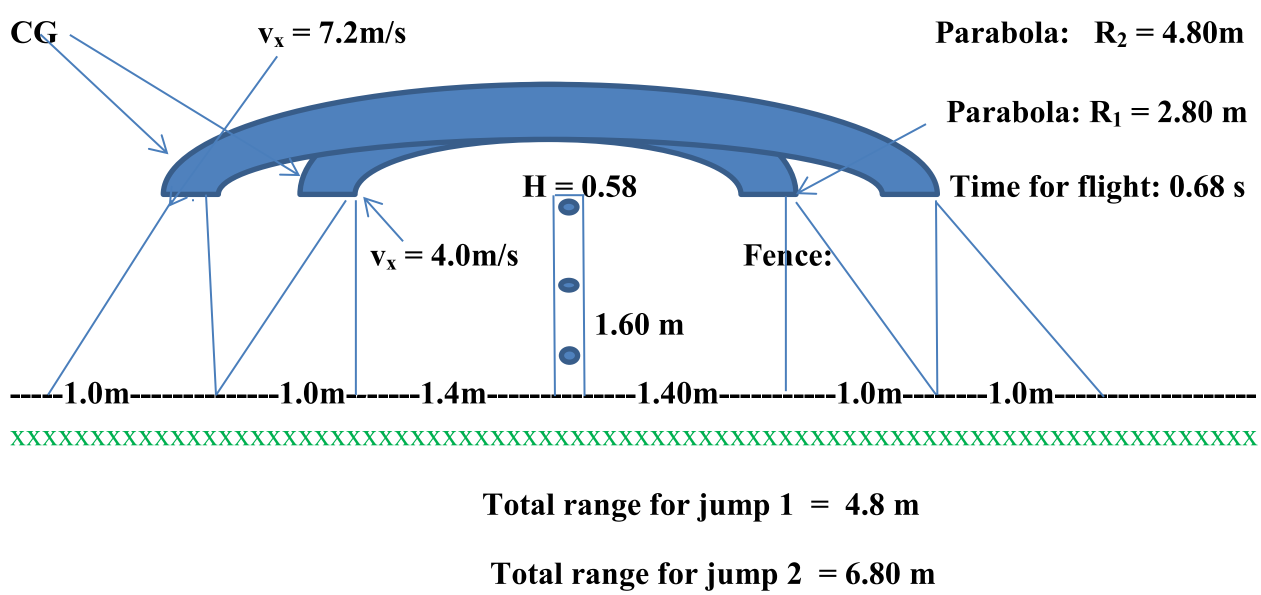


Figure 7
Comparing the kinematics of two jumps for the same height above the fence

Finally, it should be mentioned that just as in the case of the takeoff force acting behind the CG of the horse, producing a clockwise rotation, the contact force produced by the front legs on landing acts in front of the CG and produces a counter-clockwise rotation. Indeed, if the angle of descent is large enough, the horse will rotate clockwise, which results in a dangerous somersault, with a potential of severe injury to both rider and horse.

Water Jumping

If equestrian jumping is like jumping hurdles, then water jumping is similar to the long jump. Figure 8 shows an ideal water jump.

The width of the jump at the Grand Prix at Spruce Meadows was 4.2 m. So Lamaze and Derly had to make sure the jump was at least 5.0 m long (see Figure 9). The angle of elevation at takeoff was about 25° and the approach speed about 7.5 m/s, because for a range of trajectory to be 5.0 m, at an angle of 25°, the horizontal velocity must be 7.5 m/s. Following the same reasoning as before and using equations 1, 2, 3 and 4,

$$5.0 = v_x t = 7.5t.$$

Therefore, $t = 0.7$ s.

Since $\tan 25° = v_y/v_x$, $v_y = 3.5$ m/s, the height h of the trajectory is

$$h = v_y{}^2/20 = 0.61 \text{ m}.$$

The contact time for the takeoff is also about 0.2 s and, therefore, the vertical force necessary for the trajectory is given by

$$F_y = mg + m(3.5/0.2) = 570(10.0 + 17.5) = 15{,}600 \text{ N},$$

very much the same as for the 1.6 m fence.

On landing, the vertical force F_y, as for the hind legs on takeoff, is about 15,600 N. As before, the horse is reducing her speed, this time to about 5.0 m/s, from 7.5 m/s. Therefore, the horizontal force is

$$F_x = 570(2.5/0.2) = 7{,}100 \text{ N}.$$

The total force then is

$$F = (15{,}600^2 + 7{,}100^2)^{½} = 17{,}000 \text{ N},$$

a little larger than the force required for the fence jumping.

These large forces acting on the horses, even if only for a short time, are stressful for them. Riders are concerned about their horses and make sure that they are healthy, both physically and emotionally. Horses are examined by veterinarians before each competition. Serious accidents in Grand Prix jumping, unlike in steeple chasing or racing, are rare.

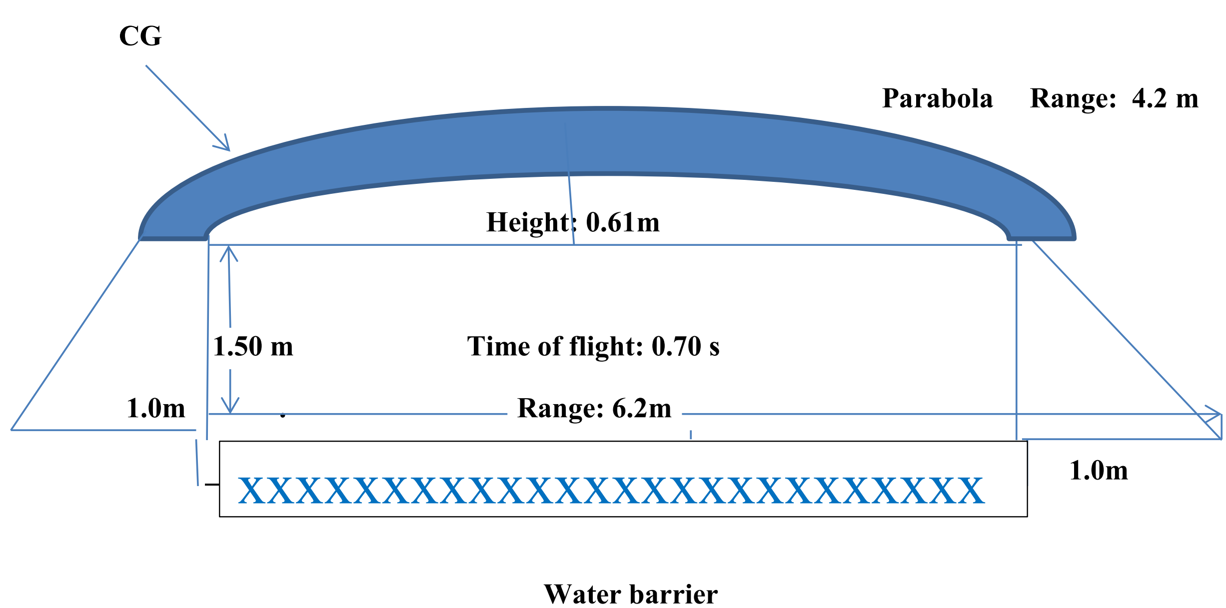


Figure 8
Kinematics of jumping the water barrier

Comparison with Hurdles and the Long Jump

The world record for the 400 m hurdles for men is about 47.0 s. That means that the average speed of the athlete in this event is about 8.5 m/s. The average speed at the Spruce Meadows International Grand Prix ring, over a distance of 550 m and with a restricted time of 84.0 s, is about 6.7 m/s. The maximum speed of a human sprinter today is about 10.5 m/s (38.0 km/h), but show-jumping horses can run as fast as 14.0–15.0 m/s (50.0 km/h). There were two places where the horse could gallop between barriers, notably before the water jump, where Lamaze seems to have allowed a speed of over 10.0 m/s for Derly over a distance of about 25.0 m.

The kinematics and dynamics of jumping over a hurdle about 1.0 m high are similar to those of the show jumper clearing a high fence. The approach speed for the hurdle, however, is much higher—about 10.0 m/s. The takeoff angle is 70–80°, almost twice as steep as that of the horse jumping a high fence. Students can work out the force required for an athlete, with his centre of gravity about the height of the hurdle and a mass of 70 kg, to clear the hurdle at a height of about 30.0 cm.

The kinematics and dynamics of the long jump are also similar to the jump of a horse over a water barrier. Olympic long jumpers typically jump over 8.0 m. They approach the liftoff point with a speed between 9.0 and 10.0 m/s. The optimum angle of elevation is about 20°. Again, students could study the kinematics and dynamics of the long jump of a world-class athlete for comparison.

Conclusion

The data used in these calculations would not be sufficient for an article in a technical research journal on biomechanics. However, our results look reasonable and the physics we used is solid, so that improved data could easily be applied. I hope that after studying the physics of equestrian show jumping, students will get more enjoyment out of watching a Grand Prix.

I hope to send a copy of this article to Eric Lamaze, and perhaps after reading it, he will invite me to Spruce Meadows to make good measurements, using Derly as our subject.[3] I would be especially interested to find out how many of these principles of kinematics and dynamics Lamaze consciously applies when judging his speed and position for jumping. However, perhaps a study of the physics of his craft would compromise his smooth and seamless riding.

Figure 9
Eric Lamaze and Derly jumping a water barrier

Notes

I would like to thank my wife, Ann, for the sketches in Figure 2 and her helpful suggestions for improving the article.

1. http://home.cc.umanitoba.ca/~stinner/teacherresources.html

2. See www.equinew.com/jumping.htm (accessed May 7, 2013).

3. Unfortunately, since the time of writing, Eric Lamaze has sold Derly and is concentrating on developing his young stallion Wang Chung M2S. He has had some good wins lately. We wish them a successful future.

References

Meershoek, L S, H C Schamhardt, L Roepstorff and C Johnston. 2001. "Forelimb Tendon Loading During Jump Landings and the Influence of Fence Height." *Equine Veterinary Journal* 33, no S33 (April): 6–10.

Moghaddam, M S, and N Khosravi. 2007. "A New Simple Biomechanical Method for Investigating Horses Jumping Kinematics." In *XXV International Symposium on Biomechanics in Sports 2007*, *Ouro Preto—Brazil,* 579–82. Also available at https://ojs.ub.uni-konstanz.de/cpa/article/viewFile/549/488 (accessed May 7, 2013).

Stinner, A O. 1980. "Physics and the Bionic Man." *The Physics Teacher* 18, no 5 (May): 358–61. (Reprinted in *New Scientist,* December 18–25, 1980, 806–09.)