The Flight of Discovery 119 March 16, 2009.

This is a draft of an LCP that is based on data given to us by NASA, describing the 'predicted' trajectory of the recent ascent of Discovery STS 119, March 16. 2009. This is only ma draft. It will be fine-tuned later.

A short version of this will be published by *The Physics Teacher*.

(It was written with the assistance of Don Metz of the University of Winnipeg)

Introduction

In Newton's thought experiment, found in his *Principia*, a cannon was placed at a height of about 25 miles (40 km). He placed the cannon at this height, thinking that the air resistance would be negligible. He showed that only one specific speed (launched tangentially) would produce a circular orbit, all others would be elliptical. Of course, we don't launch satellite that close to the earth's surface because the atmosphere, even though it has a very low density at this height, would soon slow down the satellite and it would spiral into the denser lower layers and burn up. The Shuttle made the dream of Newton expressed as a thought experiment a reality. See LCP 5.

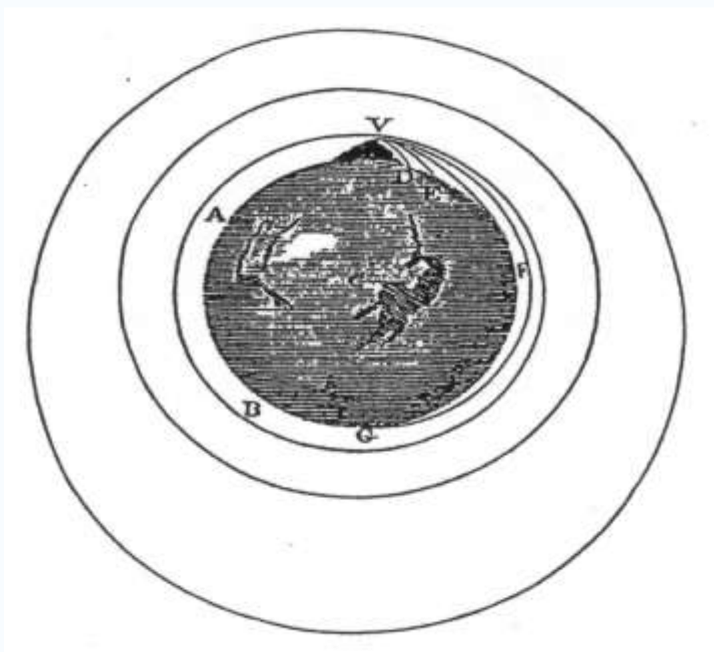


Fig. 1 Newton's own sketch in the *Principia*, (1687).

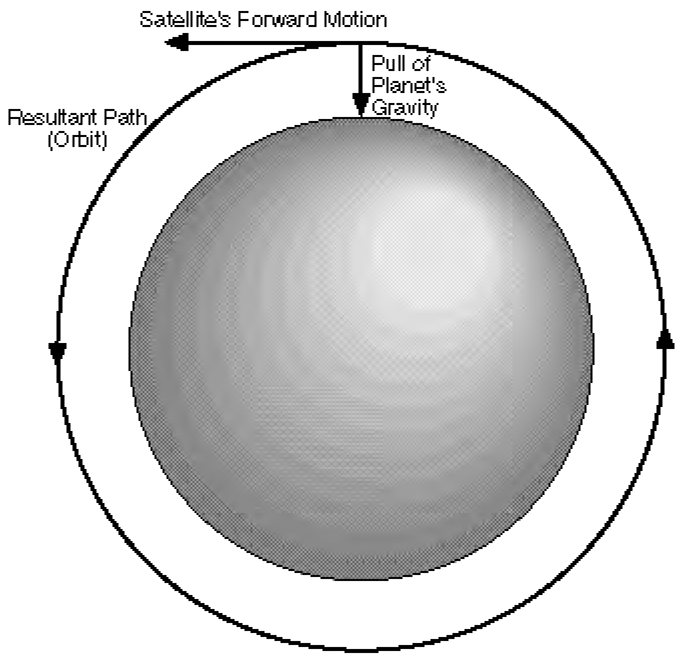


Fig.2 Centripetal acceleration produced by gravity.

Internet Links:

http://www.google.ca/search?hl=en&q=the+space+shuttle+ascent+trajectory+&btnG=Google+Search&meta=

Introduction

The Space Shuttle has been very much in the news recently and most students have seen pictures of the shape of the trajectory of a Shuttle ascent.Many are also aware of the fact that the successful descent of the Orbiter depends on using the drag of the atmosphere. However, the physics of the flight of the Space Shuttle, the ascent and the subsequent descent of the Orbiter, is often shrouded in mystery and misconceptions. Textbooks for introductory college physics generally do discuss the simple rocket equation and show how to calculate the period of the Orbiter in a circular orbit, but the physics of the trajectory of the Shuttle is seldom presented.

This LCP provides a background for a sufficiently comprehensive description of the physics (kinematics and dynamics) of the Space Shuttle Discovery launch, STS 119(Space Transport Shuttle) . The data is based on a formidable spread sheet kindly sent to us by William Horwood, who is the CBS News Space Consultant at NASA. The NASA spread sheet provides detailed and authentic information about the *prediction* of the ascent of flight STS 119, and will be the 36th flight of Discovery, the 125th Shuttle flight to date. After several postponements, the Discovery was successfully launched on to March 14 (at the time of writing this was still two weeks in the future). We have used these data for our calculations and the production of the

Using the NASA data

The complete description of the ascent of the Space Shuttle, placing the Orbiter into a preliminary orbit, is a formidable engineering task, to be sure, but it is clearly also a computational challenge. First, we had to decide what data were useful and relevant, secondly we went through the task of converting the units into the SI system, and thirdly we looked for key positions of the Shuttle's trajectory to illustrate the physics (kinematics and dynamics) of the ascent of the Shuttle.

The immediate task of finding the important columns of the spreadsheet, was difficult, mainly because the vast majority of the entries turned out to be irrelevant for our purpose. The columns we finally decided on can be seen in the two tables of data given.

The second task, that of converting the units to the SI system, was straight-forward. However, we are tempted to ask the following question: Why is NASA still using the British system of units?

For the third task, that of choosing representative points in the trajectory, we analyzed three key times, the first at the very beginning of the take-off, the second when $t = 60$ s, and the third when $t = 480$ s. The first one is important because it discusses the near static condition when forces are applied that allows the Shuttle to rise in the gravitational field of the earth. The second position is instructive because the Shuttle is entering a curved motion, significantly departing from the near vertical ascent during the first 30 seconds. This allows the analysis of the forces acting on the Shuttle as well as on the astronauts, including the force of the air drag, which at this time is a maximum.

The third and last position chosen is the time just before main engines cut off (MECO). Here the horizontal acceleration is a maximum (almost 3gs), the drag is zero, and the Orbiter (plus external tank) is moving in a near-horizontal direction. In addition, the centripetal acceleration is 8.1 m/s^2 and the astronauts are beginning to feel being "weightless", that is, they are in what NASA designates as *microgravity*. We will conclude with some suggestions for a number of additional problems.

Fig. 3. A view of the Space Shuttle Discovery at the start of the launch

Image: Space shuttle Discovery (STS119) blazes into the night sky as it lifts off Launch Pad 39A at NASA's Kennedy Space Center in Florida, March 15, 2009. Photo credit: NASA TV

Fig. 4. the ascent of the Discovery Image: Space shuttle Discovery hurtles into the evening sky on the STS-119 mission. Photo credit: NASA/Fletch Hildreth.

The launching of the Space Shuttle

The following description of the flight of the Space Shuttle is partly based on the NASA websites. However, the data given are based on the spreadsheet for STS 119. Please consult the tables for the description of the flight.

The Space Shuttle is launched vertically with all main engines firing. At an altitude of about 47 km and a range of about 50 km, after about 124 seconds, the boosters separate (SRB Staging). The Orbiter (and the large tank) continue under SSME (Space Shuttle Main Engine) power until about 8 min 50s (t=530s) after launch, when the external tank separates for destructive reentry. We will discuss the flight up to t = 514 seconds, since the NASA data given to us only go that far. The velocity reached by that time is 7581 m/s, and 7871 m/s relative to the center of the earth (this is called VI, the inertial velocity), and the range is 1471 km. It is easy to create a picture of the flight path obtained by placing the latitude and longitude for the two points into Google Earth. (See Table 1).

The following description of the flight of the Space Shuttle is partly based on the News Reference[a] manual originally distributed to the press in 1988 and the Shuttle reference data located on NASA's website[b]. However, the data we use is from a recent flight of the shuttle (Flight STS 119). It should be explicitly stated that data given to us, about six weeks before the launching, were a *prediction* and not the data of the actual flight. Please consult the tables when reading the description of the flight.

Initially, the Space Shuttle consisting of three major components, the Orbiter, the solid rocket boosters (SRB), and the external fuel tank, is launched vertically with all main engines firing. About 10 seconds into the flight, the Shuttle turns so that the Orbiter lies under the external fuel tank and the solid rocket boosters. This familiar roll is important for a number of reasons. First, it reduces the stress on the Orbiter's delicate wings and the tail of the Shuttle. Secondly, it makes it easier for the computer to control the Shuttle during the remainder of the ascent. Thirdly, it enables the astronauts to see the horizon, giving them a reference point, should the mission have to be aborted and the Orbiter forced to land. The roll ends at about 18 seconds, when the Shuttle is at an altitude of 976 m and a range of about 200 m. The velocity of the Shuttle now is 120 m/s.

The Flight of the Space Shuttle

The Shuttle then climbs along an arc, accelerating while the total mass of the Shuttle decreases. As the Shuttle continues to flatten in its trajectory, powering back the main engines to about 70% relieves stress. By about 40 seconds into the flight, the Shuttle breaks the sound barrier with a speed of about 320 m/s. About 60 seconds into the flight the Shuttle encounters the highest air resistance (drag). At this time the phenomenon known as the Prandtl-Glauert singularity occurs, when condensation clouds form during the transition to supersonic speeds. Shortly after that, however, the pressure on the Orbiter decreases and the Shuttle engines are returned to full power. At this point, the Shuttle is traveling at 454 m/s.

Two minutes (120 seconds) into the ascent, the Shuttle is about 45 kilometers above the earth's surface and traveling at Mach 4.1, or 1324 m/s. The SRBs, having used up their fuel, separate from the external fuel tank and fall back to Earth. The descent of the SRBs, some 225 kilometers downrange, is slowed by parachutes that are ejected from the nose cone marking the end of the first stage of the ascent. The second stage of the ascent begins at SRB separation when the main engines have inadequate thrust to exceed the force of gravity since the thrust-to-weight-ratio becomes less than one. However, as the engines burn fuel, the mass decreases, the thrust-to-weight ratio increases and the vehicle starts its acceleration to orbit speed. (Note: at about 228 seconds the thrust to weight ratio is over 1, see Table 2).

The second stage of the ascent lasts about six and a half minutes. With the solid rocket boosters jettisoned, the Shuttle is now powered solely by its three main engines reaching an altitude of over 104 km and a speed of 7.87 km/s, relative to the center of the earth. (See altitude-range graph). The three Space Shuttle main engines, attached to the rear of the Orbiter, continue to fire until about 8.5 minutes after liftoff. As the main engine completes its burn, the mass of the shuttle has decreased so much that that the engines are throttled back to limit vehicle acceleration to 3 g, necessary to maintain a safe and comfortable ride for the astronauts (see velocity-time and acceleration-time graphs).

Finally, about eight and a half minutes after takeoff, the Shuttle’s engines shut down and the external fuel tank is jettisoned from the Shuttle. At this time, the Shuttle is traveling at about 8.0 kilometers (5 miles) a second and the Orbiter, with a mass of 118,000 kilograms (260,000 pounds), is the last Space Shuttle component that will orbit the Earth. Note that only about 7 % of the original mass is left at the end of our description of the launch trajectory journey, when t = 514 seconds.

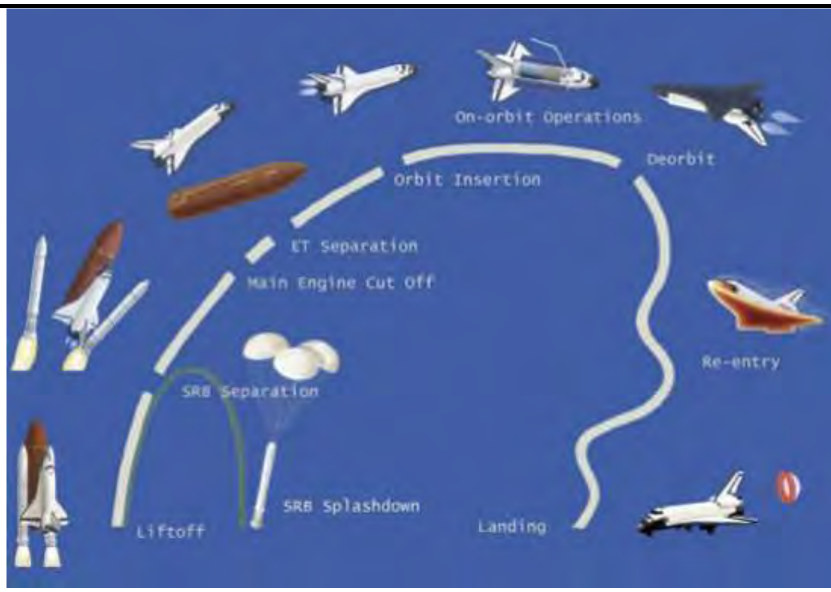


Fig. 5. The trajectory of the Space Shuttle

Fig. 6. At t = 124s SRB separation takes place.

TABLE 1

Time (s)	Alt (m)	Range (km)	θ (°)	V (m/s)	Vy (m/s)	Vx (m/s)	Accel. (m/s^2)	NASA Accel "sensed" g	α (°)	28.608N 80.604W
0	-7	0	89.8	0	0	0	4.8	0.3	90	Launch
5	46	0	89.7	24	24	0	6.0	1.6	89	
10	236	0	87.4	55	55	0	7.0	1.7	89	Start roll
15	672	0.18	71.9	97	95	19	8.5	1.8	78	
18	976	0.18	69.2	120	115	34	8.6	1.9	73	End roll
20	1211	0.18	69.5	137	129	46	8.6	1.9	70	
30	2787	0.93	67.8	220	197	97	8.5	1.8	63	
36	4032	2.0	65.9	266	234	126	8.0	1.8	61	Throttle down
40	5214	2.2	63.8	300	257	154	8.1	1.7	58	
50	7890	3.3	61.3	364	300	206	7.1	1.8	55	Throttle up
60	11380	6.5	59.1	454	363	272	10.	2.0	53	Max.AirPressure
90	25496	19.4	38.7	882	576	667	16.	2.5	40	
120	44626	47	26.1	1324	660	1147	8.1	1.0	29	
124	47341	49	25.5	1339	644	1173	8.0	1.0	28	SRB STAGING
125	48162	51	23	1341	643	1176	9.0	1.0	28	
150	63018	84	19.7	1483	545	1379	8.0	1.0	21	
180	77732	129	16.9	1696	437	1638	8.3	1.1	14	
210	89304	181	14.5	1957	334	1928	11	1.2	9	Negative return (218)
240	97930	242	12.5	2258	340	2232	11	1.3	8	
270	104006	315	10	2614	151	2609	13	1.4	3	
300	107416	397	7.8	3007	74	3006	12	1.6	1	
330	108715	491	5.5	3452	10	3451	15	1.7	0	
360	108296	600	9.4	3960	-38	3959	18	1.9	-1	
390	106808	728	25.3	4562	-62	4561	21	2.2	-1	
392	106690	737	25.2	4603	-63	4602	21	2.3	-1	
420	104776	871	22.9	5249	-70	5248	24	2.6	-1	
440	103479	980	21.1	5788	-54	5787	27	3.0	-1	
450	103016	1036	20.3	6062	-40	6061	27	3.0	-1	
480	102661	1225	17.5	6912	26	6911	27	2.9	0	
503	104059	1394	13.6	7571	98	7570	1.4	1.2	0	MECO
510	104718	1444	12.8	7581	98	7580	0	0	0	Zero Thrust
514	105077	1471	12.8	7581		7581	0	0		37.356N 68.714W

TABLE 2: THE FLIGHT O DISCOVERY 119: DYNAMICS

Time	Mass kg	Fuel loss kg/s	Thrust %	Thrust (N)	Drag Pressure (N/m^2)	VI (Inertial) (m/s)	Accel (m/s^2)	
0	2047249		100	30200000	0	408	4.8	Launch
5	2000356	-11831	104.5	31559000	364	409	6.0	
10	1939932	-12806	104.5	31559000	1833	412	7.0	Start roll
15	1866473	-12274	104.5	31559000	5342	435	8.5	
18	1829536	-12334	104.5	31559000	7948	453	8.6	End roll
20	1804824	-12361	104.5	31559000	10012	465	8.6	
30	1687000	-11182	104.5	31559000	21585	528	8.5	
36	1622233	-10440	90	27180000	27691	566	8.0	Throttle down
40	1572745	-9672	72	21744000	30816	596	8.1	
50	1479240	-9304	104	31408000	33806	656	7.1	Throttle up
60	1385150	-9573	104.5	31559000	35059	736	10.	Max.AirPressure
90	1075485	-8782	104.5	31559000	14270	1163	16.	
120	874387	-2098	104.5	31559000	1737	1617	8.1	
124	866820	-1628	104.5	5486250	1233	1635	8.0	SRB STAGING
125	695925	-1427	104.5	5486250	1230	1637	9.0	
150	657211	-1440	104.5	5486250	614	1792	8.0	
180	612567	-1440	104.5	5486250	216	2014	8.3	
210	567923	-1440	104.5	5486250	33	2278	11	Negative Return (t=218s)
240	523660	-1422	104.5	5486250	4	2580	11	
270	478122	-1424	104.5	5486250	0	2935	13	
300	434007	-1423	104.5	5486250	0	3325	12	
330	389893	-1423	104.5	5486250	0	3767	15	
360	345778	-1423	104.5	5486250	0	4270	18	
390	300240	-1423	104.5	5486250	0	4868	21	
392	297394	-1423	104.5	5486250	0	4909	21	
420	256125	-1423	104.5	5486250	0	5551	24	
440	226241	-1423	104.5	5486250	0	6086	27	
450	212436	-1335	98	5145000	0	6359	27	
480	174764	-1114	80	4200000	0	7204	27	
503	150211	-1292	60	3150000	0	7860	1.4	MECO
510	149690	0	0	0	0	7871	0	Zero Thrust
514	149690	0	0	0	0	7871	0	

Table 1:

Altitude: The height in meters, above the imaginary geodesic point at the launch sight, which is 7m (-24ft) above the center of gravity of the Shuttle at t=0.

Range: The distance in kilometers at time t, measured along the earth's curvature to the Shuttle.

Pitch Angle (P.A.): The angle from the horizontal to the Shuttle in degrees.

V = Velocity of the Shuttle at time t in m/s

$\mathbf{V_y}$ = The vertical component of the velocity of the Shuttle at time t.

$\mathbf{V_x}$ = The horizontal component of the velocity of the Shuttle at time t.

Accel. : The acceleration of the Shuttle at time t.

Accel. (NASA): The acceleration reported by NASA, as "sensed" by the astronauts.

Alpha: The angle that the tangent makes with the trajectory (See R-t graph)

Table 2:

Time: Time is given in seconds.

Mass: Mass of the Shuttle in kg, at the time indicated.

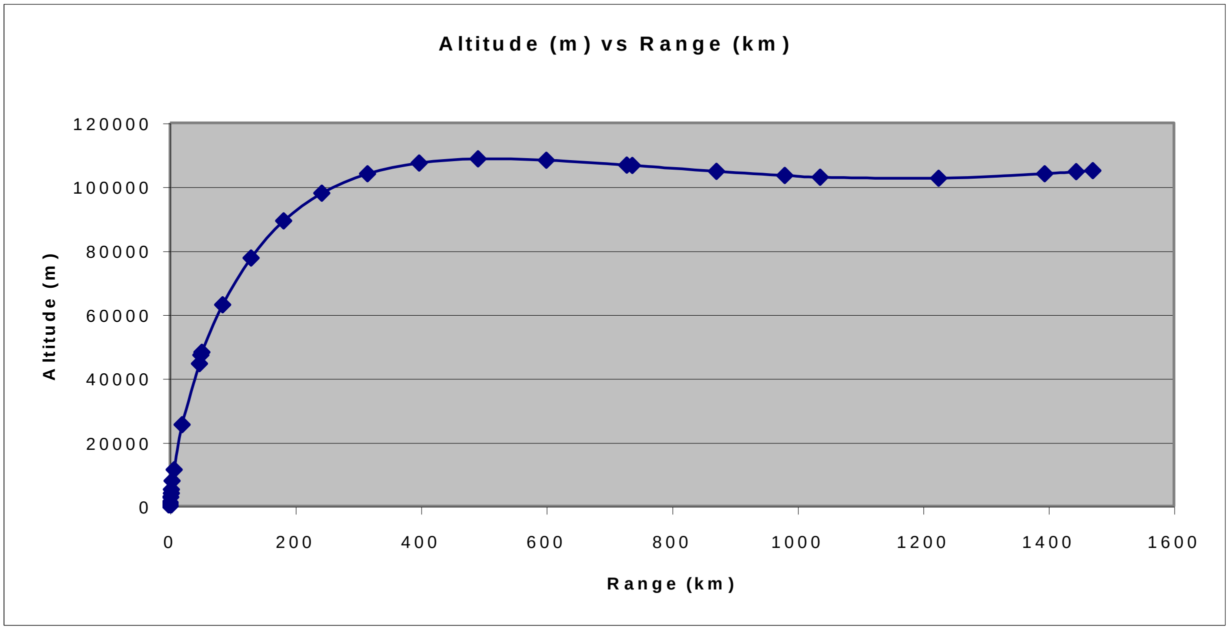
Fuel Loss Rate: The rate of fuel loss in kg/s at time t.

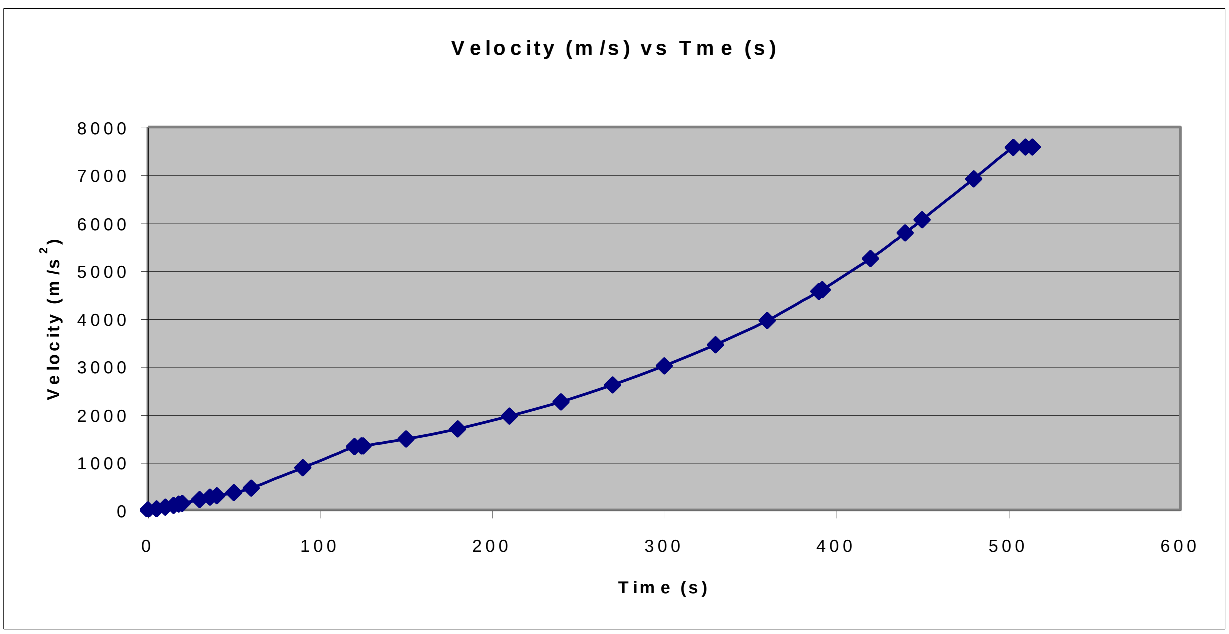
%Thrust: The value of the thrust in Newtons, based on 100% being 30200000N for the total thrust (SRB engines plus the three Orbiter engines) up to SRB staging when = 124 s. After that it is based on the Orbiter engines output of 5250000N. at 100%.

Thrust: The thrust in Newtons at time t.

Drag: The effect of the atmosphere on the Shuttle, given in N/m^2 at time t.

VI: The inertial velocity of the Shuttle, i.e., the velocity relative to the center of the earth. At the beginning of the launch, the Shuttle is already moving at 408 m/s in an Easterly direction because of the rotation effect of the earth at latitude 28.608 N. At t= 514 the inertial velocity is reduced to the


Altitude (m) vs Range (km)
Altitude (m)
120000
100000
80000
60000
40000
20000
0
0
200
400
600
800
1000
1200
1400
1600
Range (km)


Velocity (m/s) vs Tme (s)
Velocity (m/s^2)
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200
300
400
500
600
Time (s)

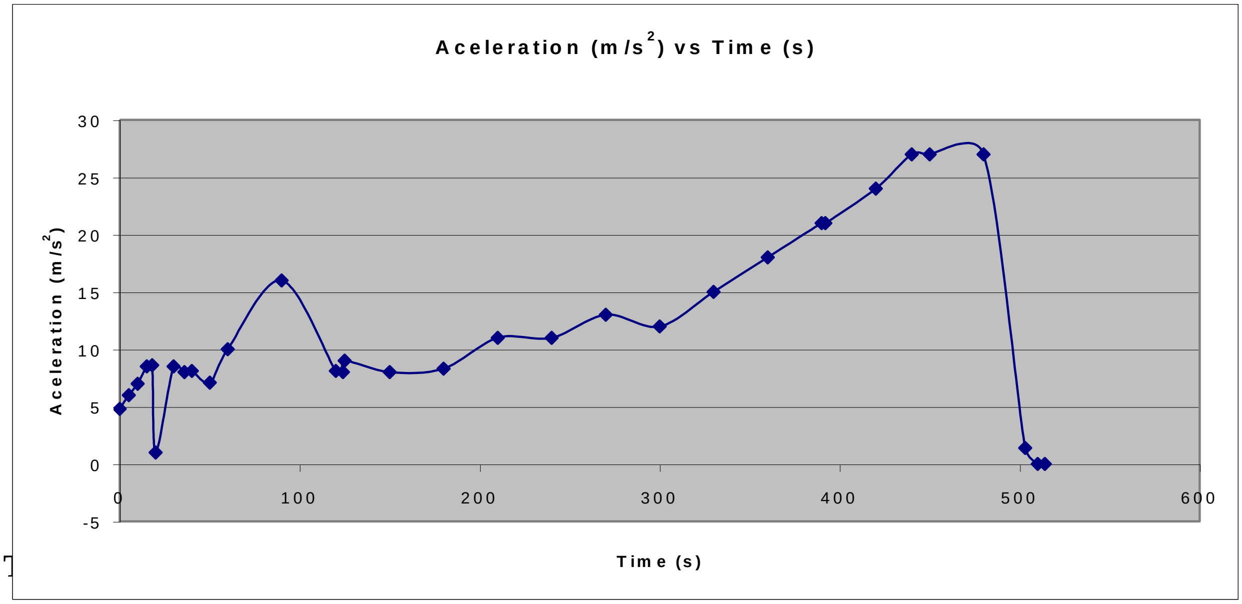


Fig. 7 . The main graphs generated: The altitude–range, the velocity-time, and the acceleration-time graphs.

Using the data given by NASA of the predicted flight of the STS 119 flight we can apply elementary physics and mathematics to understand the kinematics and dynamics of the launch. We begin with the description of the physics of the motion of the Shuttle as a function of time along the path of ascent. For each second we were given the following information about the Shuttle:

1. The altitude (m)
2. The range (km)
3. The velocity (m/s)
4. The inertial velocity (m/s) (relative to the center of the earth)
5. The pitch angle (PA) θ (degrees)
6. The vertical velocity (m/s)
7. The thrust (N)
8. The “drag” or pressure of the atmosphere (N/m^2).

We have added another parameter, the pitch angle (θ), which is the angle that the Shuttle makes as measured from the horizontal at a given time. The angle α described determines the tangent to the trajectory, or the direction of the motion of the Shuttle at any given time and is mostly different from the pitch angle. When the Shuttle begins its lift these two angles are almost equal (90 º each). During the flight they deviate slightly meaning that the Shuttle is not parallel to its motion. For example, by t = 480 s, the angle α is close to 0 º but the pitch angle is 17º.

For kinematics:

The above information allows us to calculate:

1. The angle alpha
2. The horizontal velocity of the Shuttle at any time t
3. The acceleration of the Shuttle in the x and y directions, that is, the horizontal direction and the vertical direction at given time, downrange
4. The total acceleration on the Shuttle

This will constitute the content of Table 1, which is essentially kinematics based on the data given by NASA.

Next, we will look at the dynamics of the trajectory and calculate the quantities listed below. We wish to know how well the laws of dynamics (essentially Newton's second law) can account for the kinematic results.

First, there will be a general discussion of how this is done and then we will illustrate the physics for three chosen times. The following will be calculated, using dynamics:

1. The unbalanced force acting on the Shuttle
2. The vertical and horizontal acceleration components of the Shuttle
3. The total acceleration of the Shuttle
4. The centripetal acceleration of the Shuttle for high velocities (relative to the center of the earth)
5. The acceleration "felt" by the astronauts (dynamics)

These calculations will be applied to three times:

Time 1: Just as the Shuttle lifts off the launch pad (t=0).

Time 2: At $t = 60$ seconds, when air drag is maximum.

Time 3: A $t = 480$ seconds, when the thrust is lowered, just before main engines cut off. (MECO).

Note: Our values for the acceleration were calculated for every second, based on the NASA data. The reader should refer to Table 1 (Kinematics) and to Table 2 (Dynamics) while reading the next section.

Descriptions of the three times:

1. To find the angle α:

The angle of the tangent of the motion, or simply the direction of motion of center gravity of the Shuttle, can be found by calculating the arctan of the vertical velocity divided by the horizontal velocity:

$$\boldsymbol{\alpha = \arctan(v_y / v_x)}$$

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2. <u>To calculate the unbalanced force acting on the Shuttle at any time</u>

The net force **F** acting on the Shuttle is given by the vector equation

$$\mathbf{F} = \mathbf{T} + \mathbf{D} + m\mathbf{g}$$

where T is the thrust produced by the engines of the Shuttle and **D** is the total air resistance on the Shuttle, given by **pA**, and m**g** is the weight The thrust (N) and drag per unit area **p** (N/m^2) at time **t** are given in the table. We will illustrate this when we describe the physics of the second position, when $t = 60s$.

3. <u>The vertical and horizontal accelerations components of the Shuttle.</u>

The x and y components of the unbalanced force then are:

$$\mathbf{F_x = (T – D) \cos \theta} \quad \text{and} \quad \mathbf{F_y = (T - D) \sin \theta} - m\mathbf{g}$$

where **θ** is the pitch angle, which is the angle between the Shuttle and the vertical at a given time. According to Newton's second law then we get:

$$\mathbf{a_x = (T – D) \cos \theta / m}$$

$$\text{and } \mathbf{a_y = \{(T - D) \sin \theta - mg\} /m}$$

4. <u>The acceleration of the Shuttle</u>

The acceleration can be found by using Pythagoras' theorem:

$$a = (\mathbf{a_x}^2 + \mathbf{a_y}^2)^{\mathbf{1/}}$$

5. <u>The centripetal acceleration of the Shuttle (relative to the centre of the earth).</u>

When the Shuttle reaches an altitude of about 10 km the centripetal acceleration on the Shuttle becomes significant. To determine the magnitude of the centripetal acceleration we have to use the inertial velocity of the Shuttle, that is, the velocity relative to the centre of the earth.
Before launch, the Shuttle already has a velocity of about 408 m/s East, in the direction of the latitude of 28.6ºN. You can easily show that the rotation of the earth here is 408 m/s East.

We will say that when the centripetal acceleration is 0.1 g or larger it becomes significant. At about t = 210 s the centripetal acceleration is about 0.81 m/s^2. So after this time this effect must be taken into account.

The centripetal acceleration is given by:

$$\mathbf{a_c = v^2 / R}$$

where $\mathbf{R = R_E + H}$

and **v** is the inertial velocity **VI** as recorded in Table 2.

$\mathbf{R_E}$ = Radius of the earth, about 6.37x10$^{\mathbf{6}}$ m, and

H is the altitude of the Shuttle in meters.

6. <u>The acceleration, as found in the NASA data:</u>

You may have noticed that when comparing the accelerations calculated (Table 2) and those given by NASA that the acceleration given by NASA and those we calculated are different. The reason for that is that NASA's acceleration is not the acceleration that acts on the Shuttle but the acceleration 'sensed' or "felt" by the astronauts. For example, for t = 5s, our acceleration is 6.2 m/s^2 and NASA's is 1.6 gs, or about 16 m/s$^{2.}$ Note also that the acceleration given for the very start of the lift is 0.4g. This may be due to the fact that the negative altitude number of -7 m , or-24 ft (see Table 1), is a reference to an 'idealized geodetic surface'. According to our communication with NASA, the Shuttle's center of gravity is below this imaginary surface when it is sitting on the pad.

To calculate the acceleration "sensed" is complicated by the fact that the Shuttle has a pitch angle, at times significantly less than 90º and even more tricky when we enter the region after t = 300s, when the centripetal acceleration has a significant effect.

The following may clarify the idea of "sensed" acceleration: When sitting on the surface of the earth the acceleration is zero but the sensation is "1g" whereas in free-fall acceleration the sensation is "0g". Thus, we can say that

$$\mathbf{a_{sy} = a_y + g} \quad \textbf{(seen as a vector equation).}$$

The acceleration could be measured by a spring scale placed under the astronaut, that is, "sensed" acceleration is related to normal force.

We will now suggest a formula for the acceleration "sensed" by the astronauts and then test it for the three positions we want to investigate. Let us call acceleration "sensed" a_s.

a_s has an x component and a y component:

Clearly, the x component is simply

$$\mathbf{a_{sx} = a_x}$$

and $\mathbf{a_{sy} = a_y + g + a_c}$ (Understood as a vector equation).

where $\mathbf{g = 9.8\ m/s}^2$ and $\mathbf{a_c = (VI)^2 / R}$

Therefore the acceleration "sensed" is given by

$$a_s = (\mathbf{a_x}^2 + \mathbf{a_{sy}}^2)^{1/2}$$

Discussion of the three positions of the trajectory:

1. t = 0,
2. t = 60, and
3. t = 480s

Time 1. When t = 0

The Shuttle is resting on the platform and all the engines are fired, producing a thrust of $3.02\text{x}10^7$ N after less than a second. The thrust is much larger than the weight ($2.04\text{x}10^7$ N) of the Shuttle and therefore the Shuttle begins to ascend. The force diagram here is very simple and the acceleration at that "moment" is given by

$$a = (T - mg) / m = (3.02\text{x}10^7 - 2.04\text{x}10^7) / 2.04\text{x}10^4 \text{ kg} = 4.8 \text{ m/s}^2$$

This is about 0.5g. In the NASA spread sheet we read 1.5 g. How was this figure obtained?

The acceleration "sensed" by the astronauts can be calculated using the formula suggested earlier. Prior to ascent, the astronauts "feel" the Earth's gravity and, according to the equivalence principle of inertial and gravitational masses, the effect is the same as if the Shuttle were accelerating in gravity-free space at 9.8 m/s^2.

Upon launch we only have an a_y component and the total "sensed" acceleration is :

$$\mathbf{a_{sy}} = \mathbf{a_y} + \mathbf{g} + \mathbf{a_c}$$

$$= a_y + g - 0 = 4.8 + 9.8 - 0 = 15.6\ m/s^2 = \text{about } 1.5\ g.$$

(See Tables 1 and 2).

Time 2. When t= 60s

This situation is a little more complicated. The Shuttle is climbing in a curve, the pitch angle θ is 59º and there is now a significant x-component of the thrust. Another complication is that we now have maximum air resistance of about 18% of the thrust. We have estimated the effective Shuttle area to be 167 m^2.

$A_{eff} = 167\ m^2$.

$m = 1.38 \times 10^6$ kg

$mg = 1.35 \times 10^7$ N.

$T = 3.16 \times 10^7$ N.

$p = 35059\ N/m^2$.

$D = pA = 5.85 \times 10^6$ N.

Using equations in section 3 and 4 we find that the x and y components of the accelerations are $a_y = 6.23\ m/s^2$ and $a_x = 9.62\ m/s^2$. The directly calculated acceleration (that is, the acceleration on the Shuttle) then would be $a = (a_y^2 + a_x^2)^{1/2} = 11.4\ m/s^2$. The "sensed" acceleration can be obtained this way:

Since $a_{sy} = a_y + g + (VI)^2 / R = (6.23 + 9.81 + 0)\ m/s^2$,

then $a_{sy} = 15.3\ m/s^2$

So that $a_s = (a_{sy}^2 + a_x^2)^{1/2} = 18.1\ m/s^2$, or about 1.9 g

The acceleration reported by NASA for this position is 2.0 g.

In which direction is the Shuttle moving?

Earlier we argued that $\alpha = \arctan(v_y / v_x)$

Substituting the values, taken from Table 1, we get

$$\alpha = \arctan(363/272) = 53°.$$

The Shuttle at t = 60s then is pointing at about 59º (angle θ) from the horizontal but the motion is tangent to the curve at 53º (angle α).

Time 3. When t = 480s.

The Shuttle (the Orbiter plus the large tank) is pointing at a pitch angle of 17º, but moving almost horizontally at 7204 m/s (inertial velocity) and at a high acceleration. Therefore, the angle α is close to zero and the air drag is negligible.

$T = 4.20\text{x}10^6$ N

$m = 1.75\text{x}10^5$ kg.

VI = 7204 m/s.

The horizontal acceleration is:

$$a_x = T_x / m$$

and $T_x = T\cos 17° = 4.20\text{x}10^6 \cos 17° = 4.02\text{x}10^6$ N.

Therefore $a_x = 4.02\text{x}10^6 / 1.75\text{x}10^5\ \text{m/s}^2 = 23\ \text{m/s}^2$`

The vertical acceleration is:

$$a_y = (T_y - mg + mv^2/R)/m = \{(T_y - m(g - v^2/R)\} / m.$$

Note that at this high velocity the centripetal acceleration becomes a factor. For v we use the VI value of 7204 m/s. Clearly, when v2/R = g the astronauts are in microgravity

Then $T_y = 4.20\text{x}10^6$ N, and

$$a_y = \{4.20\text{x}10^6 - 1.75\text{x}10^5(9.7 - 8.1))/ 1.75\text{x}10^5$$

Therefore $a_y = 1.6\ \text{m/s}^2$

The acceleration on the Shuttle then is:

$$a = (a_y^2 + a_x^2)^{1/2} = \{(23^2 + 1.6^2)^{1/2} = 23.2\ \text{m/s}^2 = 2.4\ \text{g's}$$

NASA has an acceleration of 2.9g.

One could discuss the reason for this difference. For example, one wonders about the accuracy of the pitch angle given for this position. Note that the Shuttle is accelerating somewhat vertically at this point, as you can see in Table 1

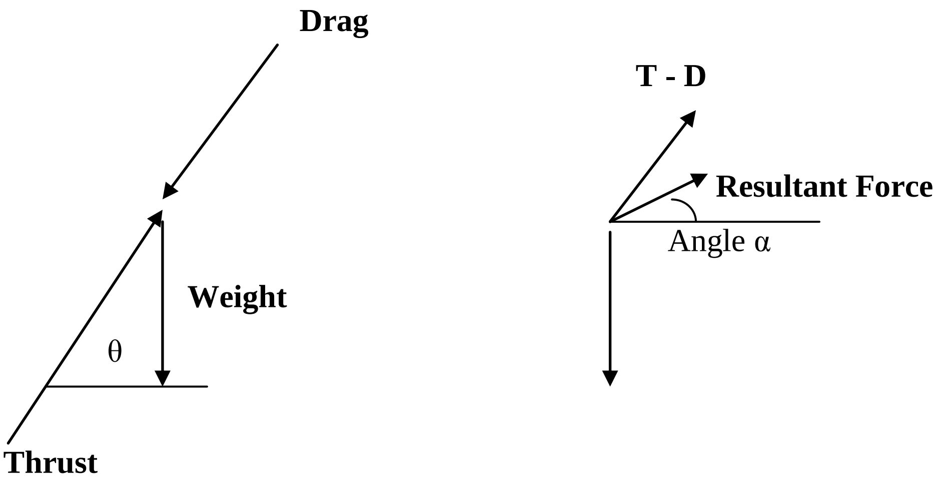
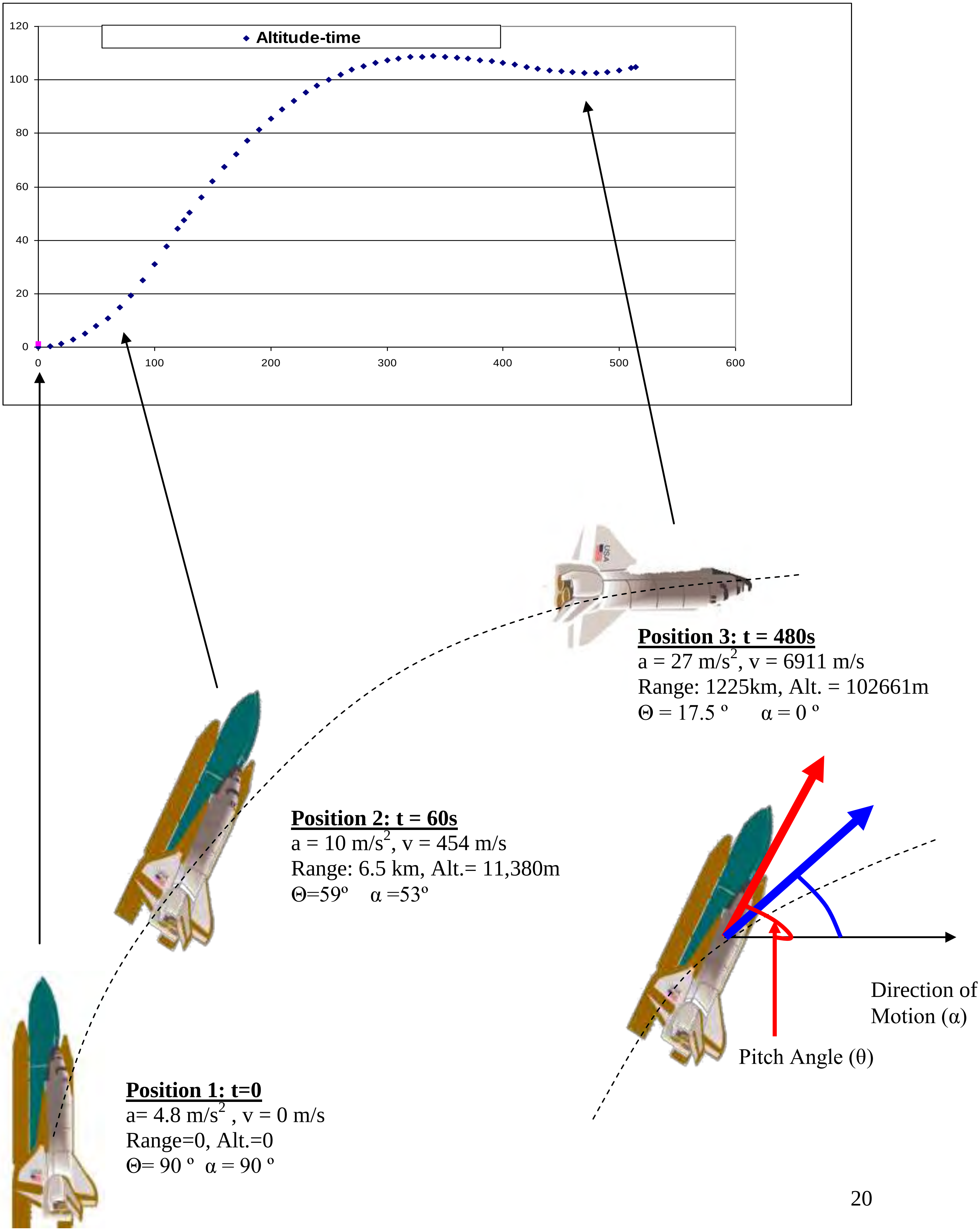


Fig. 8: **The general force-diagram for the flight of the Shuttle.**

Fig. 9:The three times (positions) of the Shuttle discussed.



It may be instructive to also discuss the relationship between the thrust and the rate of mass loss at a given ejection velocity. The data for the Space Shuttle is as follows:

1. For the two SRB engines:

    The ejection velocity: 2800/s

    The ejection rate: 8400 kg/s

2. For the three engines of the Orbiter:

    The ejection velocity: 3850 m/s

    The ejection rate: 1430 kg/s

The thrust in Newtons is given by Rv. So the combined thrust at the start (t=0) is:

$$T = 2800x8400 + 3800x1400 = 2.9x10 \text{ N}$$

From Table 2 we see that the thrust is $3.02x10^7$ N.

Some advanced problems

We thought that it would be interesting to present a number more advanced problems for teachers and interested students.

Problem 1:

Using only the latitudes and longitudes for the Shuttle trajectory up to t = 514 s, calculate the range of the Shuttle, as indicated in Table 1.

The latitudes and longitudes are (taken from the NASA spread sheet):

28.608 N and 80.604 W for the launch site.

37.356 N and 68.715 W for the point vertically below the Shuttle at t = 514a.

The Flight of the Space Shuttle

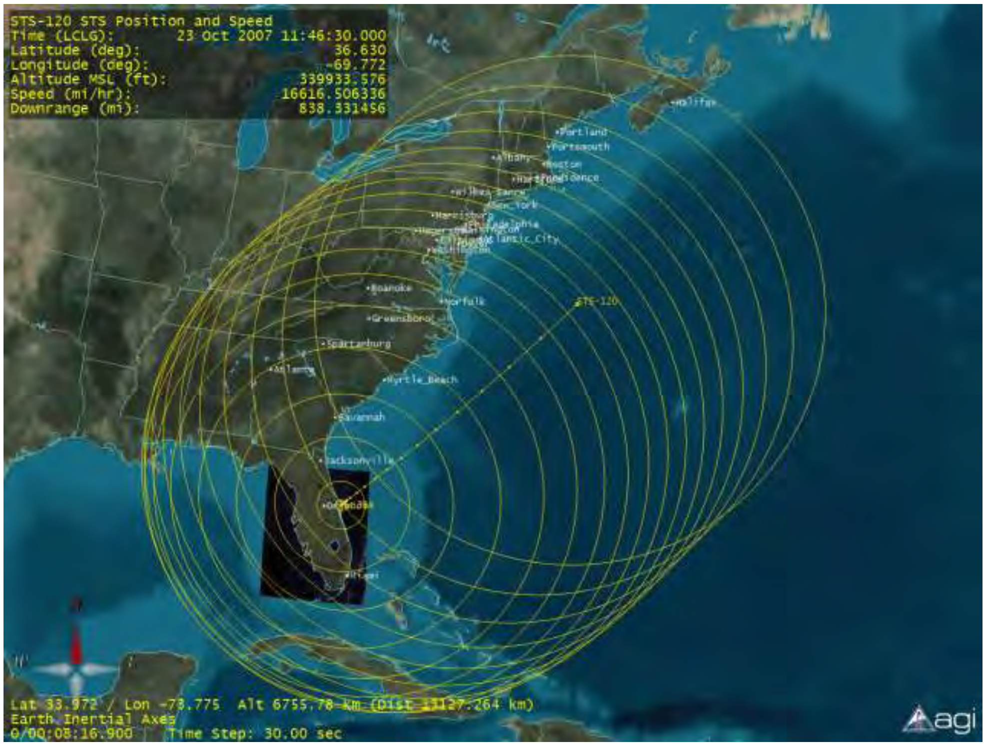


Fig. 10: The flight path the Shuttle STS 129, obtained from Google Earth. This trajectory is almost identical to that of Discovery STS 11

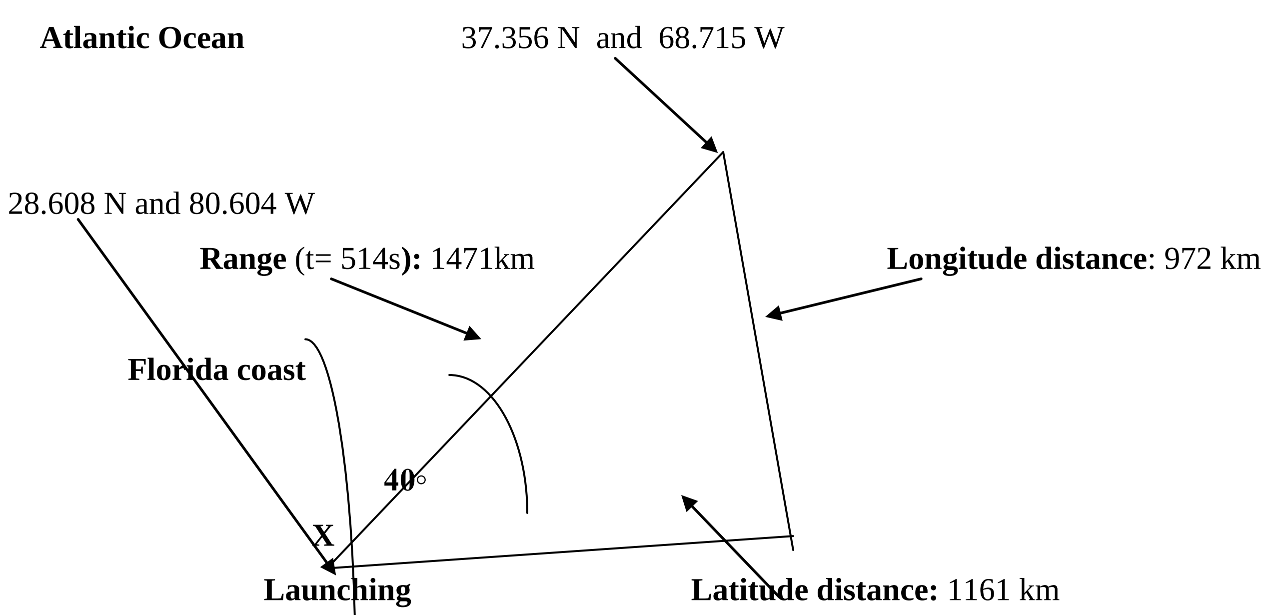


Fig. 11 A diagram to show the range calculation

An approximate solution:

Measuring along the latitude of 28.608ºN we find that the "angular distance" between the two longitudes is 11.89º and the angular distance between the two latitudes, as measured along the longitude of 68.715W is 8.74 º . Along the equator and along the any longitude, each degree is equivalent to a distance of 111.2 km. The latitude along which we are measuring the distance is parallel to the equator at 28.608 º. Therefore, the distance along this latitude for every degree will be 111.2 km x cos 28.608 = 97.62 km.

The distance along the latitude: 97.62 x 11.89 = 1161km

The distance along the longitude 68.715: 111.2 x 8.74 =972 km.

We will assume a flat earth for the calculation of the range and use Pythagoras' theorem:

$$R = (1161^2 + 972^2)^{1/2} = 1514 \text{ km.}$$

The range given by NASA is 1471km. The percentage error here is 2.8%. A pretty good result for an approximate calculation.

To solve this problem rigorously requires a formula based on spherical trigonometry. See last section.

Problem 2:

Use the simple rocket equation to find the change of velocity of the Shuttle (Orbiter plus large tank) travelling from t = 270s to t = 420s (See Table 1 and Table 2). From the tables we have:

1. The altitudes: 104006 m, and 104776m, respectively
2. The speeds: 2614 m/s and 5788 m/s.
3. The masses: 478122 kg and 256125kg.

Using calculus (see, for example, *Fundamentals of Physics*, by Halliday and Resnick) it can be shown that**,** in gravity-free space the velocity a rocket is given by

$$\mathbf{v = v_e \ln (m_i/m_f)}$$

where $\mathbf{v}$ is the final velocity (having started from rest) **,**$\mathbf{v_e}$ the ejection velocity (relative to the rocket), $\mathbf{m_i}$ is the initial mass of the rocket (plus fuel), and $\mathbf{m_f}$ the final mass at time t. This is considered the most important equation of rocket propulsion.

However, for this portion of the trajectory, the Shuttle is travelling in a nearly constant gravity (and negligible air drag) in a roughly horizontal direction. We will also assume that the resultant vertical unbalanced force on the Shuttle is very small and negligible (See the discussion of position 3 when t = 480 s). So the simple rocket equation should apply if we only want an approximate answer**.**

We can estimate the ejection velocity by calculating the average rate at which gases are ejected, using Table 2. This rate is almost constant at 1423 kg/s. Since the product of rate R (kg/s) and the ejection velocity (m/s) is equal to the thrust **($T = R\, v_e$)**, we can calculate the ejection velocity.

$$\mathbf{v_e = T / R} = 5.486\text{x}10^6 \text{ m/s} / 1423 = 3855 \text{ m/s}.$$

Applying the rocket equation we use m_i = 478122kg and m_f = 256125kg.

We can now find the change in velocity **Δv** between these two points, if the Shuttle travelled horizontally:

$$\Delta v = v_e \ln (m_i/m_f) = 3855 \ln(478122\text{kg} / 256125\text{kg.}) = 2474 \text{ m/s}.$$

From Table 1 we find that the change in velocity is actually 2635 m/s. The discrepancy here is about 6%.

Problem 3:_Check the following statement:

Taken from: website address:

http://en.wikipedia.org/wiki/Space_Shuttle_Solid_Rocket_Booster

The recovery sequence begins with the operation of the high-altitude baroswitch, which triggers the pyrotechnic nose cap thrusters. This ejects the nose cap, which deploys the pilot parachute. Nose cap separation occurs at a nominal altitude of 15,704 feet (4,787 m) about 218 seconds after SRB separation

Water impact occurs about 279 seconds after SRB separation at a nominal velocity of 76 ft/s (23 m/s). The water impact range is approximately 130 nautical miles (241 km) off the eastern coast of Florida.

Using high school kinematics, and not taking account of air resistance in the lower regions of the atmospher, we can calculate the following:

1. The height to which the free SRBs rise: about 14 km above the separation height. The hight above the ocean they rise to is about 61 km.

2. The total time before the rocket would splash down if the parachuts did not open: 165s)
3. The distance the rocket (along the line of sight of the Shuttle trajectory) will be about 203 km from the point of separation.
4. The distance from the launching pad: about 253 km

Our calculation shows that the distance from the launch pad where the splashdown takes place is about 253 km, which is very close to the one reported above.

B = Separation of SRBs. **A = Launch site**

C = Highest point reached by SRBs.

E = Point of splash down **Parachute opens at a height of 4800m**

Total height reached **Total distance from Launch A.**

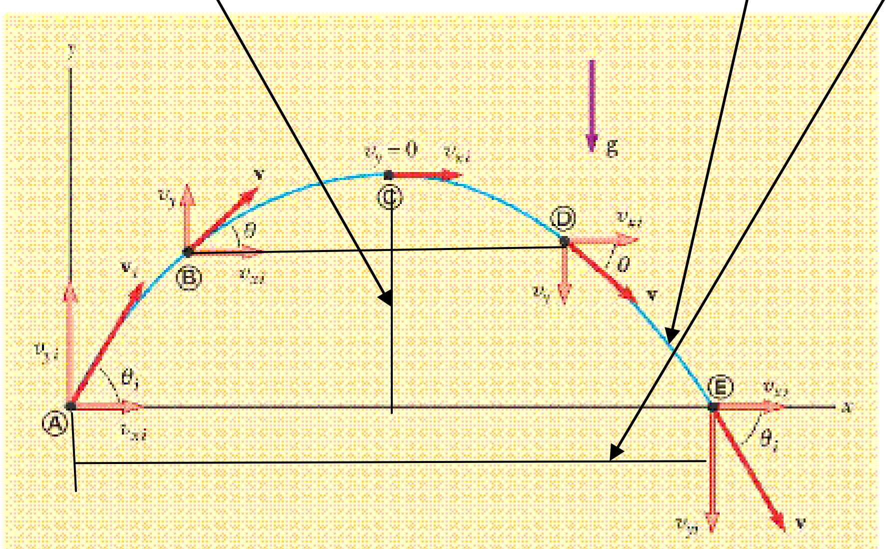


Fig. 12: For problem 1.

Suggested problems and research for students:

1. Given the latitude and longitude of the launch and the position of the Shuttle at t = 514s, use Google Earth to enable you to visualize the trajectory of the Shuttle.
2. To find the distance of the trajectory with a high accuracy requires a formula based on spherical trigonometry. The distances are measure in angles (radians). You can use the cosine law for a spherical triangle (See Fig.) :

$$\cos a = \cos c \times \cos b + \sin c \times \sin b \times \cos A$$

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You can show that the cosine law for spherical trigonometry reduces to the familiar cosine law of trigonometry, as you would expect.

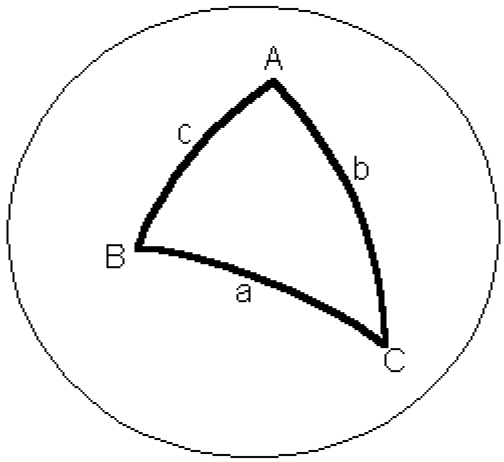

Fig. 13 A picture of a triangle in spherical trigonometry

3 a. What would be the velocity of the SRB shells just before hitting the water if there were no air resistance?

b. Find the terminal velocity of the SRB shells given the equation: of the terminal velocity of a heavy object:

$$\mathbf{V_t = \{2\ mg / C_D\ \rho A\}^{1/2}}$$

ρ = Density of air (kg/m^3), V_t = terminal velocity (m/s) , A = Area (m^2) and

C_D = Coefficient of drag (Dimensionless).

Take the area of the SRB (effective area of contact) as 40 m^2, C_D as about 0.5. and the density of air about 0.8 kg / m^2 .

Coriolis acceleration has a significant effects the motion of the Shuttle, especially after t=300s, when the velocity component in the *northerly direction* becomes high. This motion can be discussed in class. A guided approach is as follows:

The acceleration of the Shuttle, due to the rotation of the Earth, is given by the Coriolis acceleration formula (see a college level text) as :

$$\mathbf{a_c = 2\ \omega\ x\ v_r}$$

where $\mathbf{a_c}$ is the centripetal acceleration, $\boldsymbol{\omega}$ the earth, and $\mathbf{v_r}$ is the velocity along the radius of the imaginary disc that is revolving with an angular velocity equal to that of the of the earth. Students should ask their your instructor to explain this vector notation. (Students have already encountered a similar vector relationship when discussing the nature of magnetic field direction, current direction and the direction of the force acting on the current)

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a. Calculate the Northerly velocity component (along the longitude at the time of the Shuttle reaches a velocity of 7581 m/s, t= 514s. Show that this is 7581xcos50 = 4873 m/s.

b. Show that the resultant instantaneous velocity along the radius of the imaginary disc of the earth, v_r, is 4873xcos(90-37)= 2933 m/s.

c. Show that the coriolis acceleration for t= 541s then is given by:

$$a_c = 2v_r\,\omega = 2x2933x7.3x10^{-5} = 0.43\ m/s^2\ .$$

d. In which direction does this acceleration act on the Shuttle?

Is this a significant acceleration? It is significant when you consider that after only 514s the Shuttle moves in the Westerly direction by about 30 km. Using the NASA Excell spread sheet you can calculate the Westerly displacement for every second. Then calculate the total displacement of the Shuttle at the end of our considered trajectory at t= 514 s:

$\mathbf{\Delta x = a_n (t_n^2 - t_{(n-1)}^2)}$ To find total displacement x add up all the terms:

$x = \Sigma\, \mathbf{a_{cn} (t_n^2 - t_{(n-1)}^2)}$, from n=1 to n= 514).

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http://daphne.palomar.edu/pdeen/Animations/34_Coriolis.swf

A good applet illustrating the Coriolis effect.

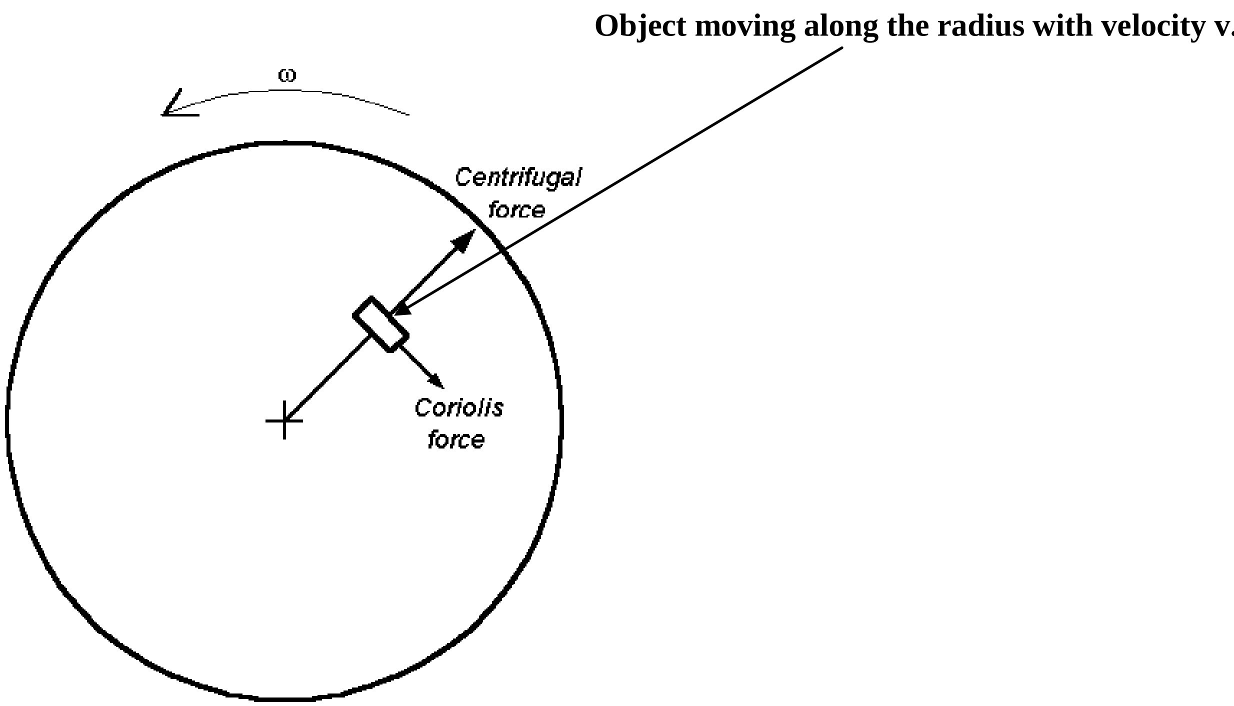

Fig. 14: The direction of the Coriolis force : $\mathbf{a_c = 2\, \omega \times v_r}$

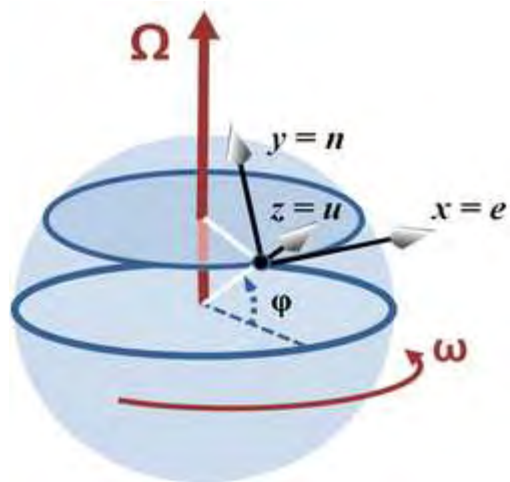

Fig. 15: The vector nature of the cross product for coriolis acceleration.

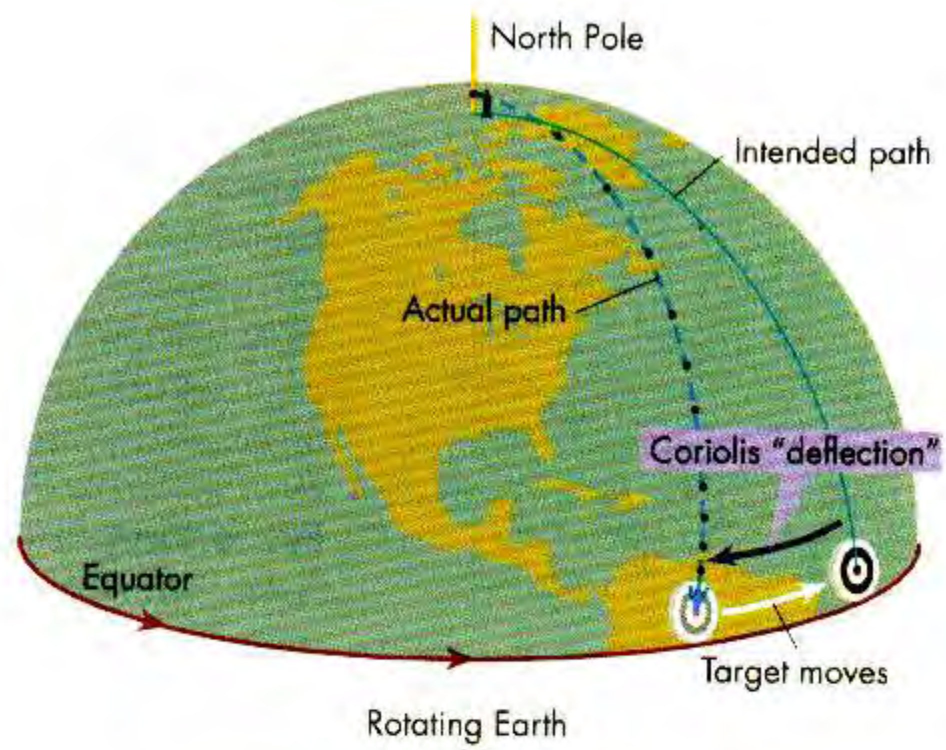

Fig. 16: A simple example of Coriolis acceleration

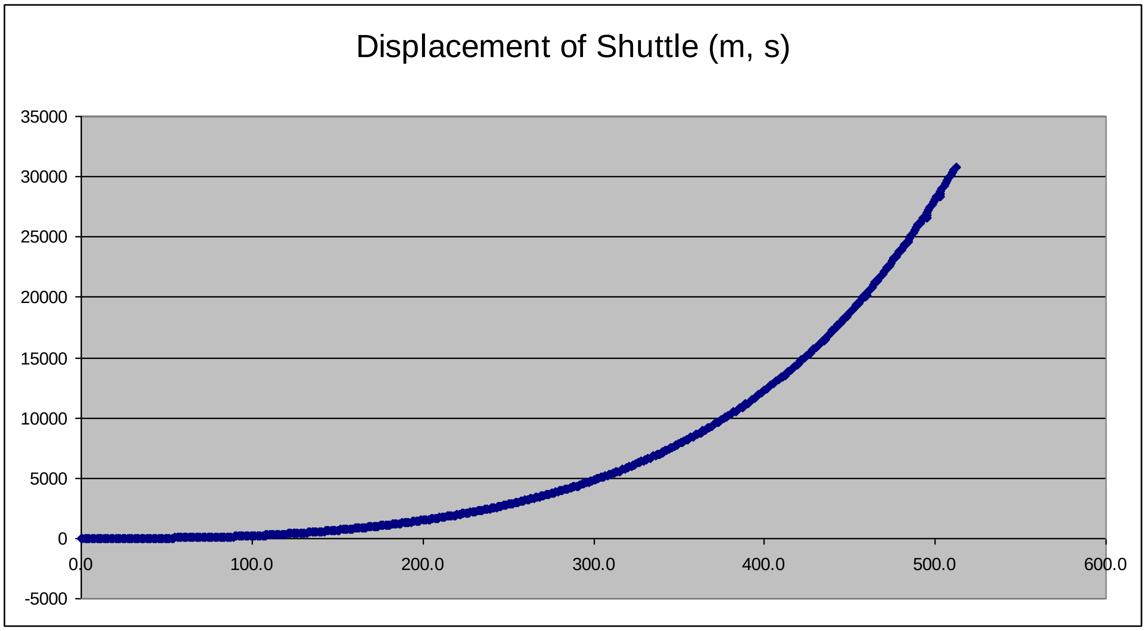

Fig. 17 The displacement of the Shuttle per second due to coriolis acceleration.

Conclusion:

In this description of the predicted flight of STS 119 of the Shuttle Discovery, we used original and complete data given to us by NASA. The tables in our article provide information, albeit for only a restricted number of positions along the ascent trajectory. However, we want to restate that the x and y components of the velocities, the accelerations, and the fuel rates of consumption were all calculated directly from the NASA spread sheet for every second.

Our main objective in this article is to give physics teachers of secondary schools and colleges a good basis from which to present the story of the launch of the Space Shuttle, in whole or in selected parts. The concepts and the level of the mathematics required to follow the arguments should be accessible to all physics teachers. The challenge of the physics instructor then is to find ways to present the story or part of the story in a comprehensible form that is appropriate to the students' mathematical and conceptual understanding. This is a good science story told in a rich context that will surely attract the attention of all students and clear up many of the common misconceptions held about the flight of the Space Shuttle.

References:

Marion, J, and Thornton, T. (1988). *Classical Dynamics*. HBJ Publishers, New York

NASA Space Shuttle reference. Taken from: http://www.braeunig.us/space/specs/Shuttle.htm (March, 2008).

The Space Shuttle description of the three stages of the flight, taken from : http://www.stemnet.nf.ca/CITE/sts_ascent.htm (March, 2008).

NASA Table of Trajectory, taken from http://spaceflightnow.com/Shuttle/sts114/fdf/114trajectory.html (March, 2008).

NASA Space Shuttle Launch (2005), taken from http://en.wikipedia.org/wiki/Space_Shuttle, (March,2008)

NASA Solid Rocket Booster Jettison, taken from:

http://www.aerospaceweb.org/question/spacecraft/q0183.shtml (March, 2008).

NASA Table for the trajectory (2005), taken from: http://spaceflightnow.com/Shuttle/sts114/fdf/114trajectory.html (March, 2008).

NASA report on Atmosphere and Spacecraft Reentry, taken from: **www.aerospaceweb.org/.../spacecraft/q0218.shtml** (March, 2008).

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http://www.cbsnews.com:80/network/news/space/119/119ascentdata.html

CBS NEWS STS-119 Trajectory Timeline

Specs for the Space Shuttle:

http://en.wikipedia.org/wiki/Space_Shuttle

Specs for Orbiter:

Calculations of drag:

http://www.physicsforums.com/archive/index.php/t-189794.html