Archimedes and Aristarchus on Samos. DRAFT

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Aristarchus and Archimedes meet on a beach on the Island of Samos in the year 245 BC. Aristarchus is about 65 years old and Archimedes about 40. In the sky we see a full Moon. Archimedes is carrying a small box full of sand and a long stick: he is walking slowly while thoughtfully testing the fine sand. He stops and draws figures in the sand. It is possible to see that he is drawing the outline of a mountain with what looks like a tunnel running through it. He draws lines that end up looking like a triangle. He studies them. He nods and then shakes his head.

Meanwhile, we also see a second figure, some distance away, standing, holding up a plumb bob against the Moon. It s clear, even from a distance, that the plumb line of the string neatly divides the Moon into two halves. He nods and then walks over to the bent-over figure of Archimedes. Archimedes seems oblivious to his presence.

Aristarchus appears and Archimedes looks up, a little startled.

Aristarchus:
Good evening, Sir. I presume you are Archimedes, the mathematician from Syracuse.
Archimedes continues his reflections and motions Aristarchus to be quiet.

Archimedes:
I do not want to disturb my train of thought.
He continues moving his stick for a while, then he abruptly stops, and looks up.

Good evening.
Looks at Aristarchus for a few seconds.

Yes, I am he.

Aristarchus:
I heard that you were on the Island and a friend of mine suggested that you were probably on the beach, thinking about geometry.

Archimedes:
Actually, I do my best work walking along a beach in the evening, occasionally stopping to draw figures in the sand, ….when nobody disturbs me and my circles.
He emphasizes the last part. He looks at Aristarchus. They both laugh.

Aristarchus:
He looks at the figures Archimedes drew in the sand.

I am an amateur geometer myself.

Looks at the figures in the sand again and smiles.

I believe you are studying the geometry of our famous tunnel, the tunnel of Eupalinos.

Archimedes:
Indeed! I am trying to understand how Eupalinos managed to work out the direction and the distances for the tunnel about three hundred years ago, digging from both ends at the same time.
He looks at Aristarchus

Aristarchus:
That must have been a challenging problem three hundred years ago.

Archimedes:
It is still an interesting problem in geometry, not to mention the engineering difficulties they must have encountered.

Aristarchus studies the drawing in the sand.

Archimedes:
Since you claim to be an amateur geometer, let me explain to you how I think he probably have used geometry before starting to dig the tunnel.
He looks at the figures briefly and then continues to explain.

Aristarchus:
That is fascinating, Archimedes. But, …
He continues and then questions one of Archimedes' guesses how the tunnel was built. Archimedes looks at Aristarchus who is still holding his plumb bob, then looks at the Moon and smiles.

Aha. You are too modest my good Sir, ….you must be the one they call Aristarchus, the mathematician, whose work I recently studied with great interest.

Aristarchus:
I admit, I am he.
But compared to you, I am mostly an astronomer.

Archimedes:
And a very good one. Of course, I intended to meet you. In fact, one of my objectives in coming come to Samos was to meet you and discuss with you your method of measuring the distance to the Moon and the Sun. But I am still not clear about a few details of your calculations and I want to ask you some questions.
He smiles.

And also discuss your 'revolutionary', shall I say daring, idea that the Earth revolves around the Sun.

It is a great honour for me to finally meet you, Aristarchus.

Aristarchus:
The pleasure is all mine, Archimedes.
I have read your brilliant work, the Sand Reckoner. In finding the upper bound for the size of the Universe you used my measurements of the distance to the Moon and the Sun and challenged my idea that the Earth revolves around the Sun.
He smiles…

Unfortunately, I was not there to respond to your challenges.

Archimedes:
Well, you are here now. Maybe I have to write a new version of the Sand Reckoner after our discussion tonight.

Aristarchus:
I don't think so. I trust your mathematics,--it is your astronomy I may criticize.

Archimedes:
Good. In all modesty, I must say that mathematicians have in the past criticized my work, but, as hard as they tried, were not able to find my theorems wanting.
He smiles and then continues.

But in astronomy I must bow to your knowledge.

Aristarchus:
Thank you, Archimedes, I am flattered.
He looks at the box of sand that Archimedes is carrying.

Are you still counting the number of sand particles in the universe?

Archimedes:
This box of sand is just a constant reminder of the need to understand, describe and measure the Universe.
He seems very serious.

In my work I address King Gelon, who, like everyone else, thought that the number of the sand particles is, if not infinite in multitude, then uncountable. And I mean by the sand not only that which exists about Syracuse and the rest of Sicily but also that which is found in every region whether inhabited or uninhabited, including this lovely beach.
Hel looks at Aristarchus.

And in the whole universe!

Aristarchus
Well, I myself always thought, before studying your work, that this number would be uncountable.

Archimedes:
Well, at least I managed to convert two persons.
They laugh.

Aristarchus:
If I remember correctly, you then referred to my estimates of the size of the universe and also mention my ides that the Earth revolves around the Sun.

Archimedes:
I welcomed and agreed with your calculations of the distances but could not agree with your second idea, the hypothesis that the Earth moves around the Sun.

Aristarchus:
I hope I will be able to convince you that my second idea, that you called a "hypothesis", is also a reasonable one.

Archimedes:
I could not accept the hypothesis that the fixed stars and the Sun remain unmoved, that the Earth revolves about the Sun in the circumference of a circle, the Sun lying in the middle of the orbit.
He stops for a moment.

Aristarchus:
Archimedes, can you give me a good reason for rejecting this simple, but perhaps unconventional idea?

Archimedes:
The main reason is that this system would lead to an immeasurably large universe.
And I argued that this is so because your hypothesis implies that the sphere of the fixed stars, situated about the same centre as the Sun, is so great that the circle in which you suppose the Earth to revolve bears such a proportion to the distance of the fixed stars as the centre of a sphere bears to its surface.

Aristarchus:
I see. Do you think King Gelon understood your argument?

Archimedes:
Probably not.
They laugh..

Aristarchus:
Let us then ignore the details of your arguments against my hypothesis in the Sand Reckoner for now. What basically did you try to accomplish in the Sand Reckoner?

Archimedes:
Agreed, we can discuss your hypothesis late.
Well, I wanted to find a counting system that allowed me to estimate the number of sand particles that would fill the whole Universe. As my smallest particle I took a fictitious sand particle that would have a constant size equal to that of a poppy seed.

Aristarchus:
Why a poppy seed?

Archimedes:
Well, a poppy seed is familiar to everyone and it is a small a particle that we can clearly see and still measure.

Aristarchus:
I will accept that explanation for now.
But I was flattered to discover that you had chosen the size of the Universe based on my calculations to estimate the distance to the Sun.

Archimedes:
I was very impressed by your measurement of the distance to the Moon and especially to the Sun. I used your ideas about the size of the Universe and then, but just to be sure, I made the universe a little bigger, in order to find the upper bound for the diameter of the Earth, the distance to the Sun and the size of the Universe itself.

Aristarchus:
You took the distance to the Sun as 30 times the distance to the Moon as the upper bound.
My upper bound was 20. Did you actually repeat my measurements?

Archimedes:
No, of course not. I am essentially a mathematician, not an astronomer.
He stops for a moment.

However, I did find a method of measuring the angular size of the Sun and the Moon and found it to be about ½ a degree, disagreeing with the 2 degrees you mention in your work.

Aristarchus:
I see. Well, this was an estimate I made a long time ago. I later revised that and my recent measurements agree with yours.
He points to the stick Archimedes is carrying.

May I borrow your stick, Sir.
Archimedes hands Aristarchus the stick.

It is very easy to make a quick estimate of the angle of the Sun and of the Moon. What I have done is use a much longer straight wooden plank, and placing on it securely a very round large coin at the end, holding it against the Moon, and then finding the length of the plank, when the large coin just covers the full Moon.
He places a small coin in a groove that he cuts at the end of the stick and holds it up.

You see, I can now measure the ratio of the length of the stick to the diameter of the coin. Doing this carefully and measuring it many times, with a much longer wooden plank and a larger coin, you would find that the ratio is always about 110.

Archimedes:
Very ingenious. That means that the ratio of the distance to the Moon to its diameter is about 110.

Archimedes:
And, clearly, this ratio applies also to the sun.

Aristarchus:
Yes. But I found that it is dangerous to look at the Sun for a long time.
(Of course, we know that the apparent sizes of the Moon and the Sun are the same from viewing a total eclipse).
He stops for a moment

Anyway, this allows you to find the arc made by the coin to a circle of the size of the stick as about 1/720, which is about ½ a degree of angle.

Archimedes:
Good. That is settled then.
But how did you measure the distance to the Moon?

Aristarchus:
I am sure, you have seen a lunar eclipse, when the Earth, the Moon and the Sun are in line?

Archimedes:
Yes, I have seen one in Alexandria recently when I visited the library there and the young librarian and mathematician Eratosthenes. Later, I must tell you an idea he has about measuring the size of the Earth.

Aristarchus:
Yes, I have heard rumors about that. That would be an important discovery.
Anyway, as the Earth's shadow moves across the Moon, it can be determined that the radius of the shadow is about 2 and a ½ times the radius of the Earth. Knowing this and

the fact that the angle of the Sun's as well as the Moon's circumference, as seen from the Earth is the same, we can estimate the distance between the Earth and the Moon in terms of the diameter of the Earth.
He stops for a moment then continues to explain it drawing two circles in the sands, one for the Earth and one for the Moon, and connecting them with two lines inresecting somewhere in space with an angle of ½ a degree.

The distance to the Moon then is about 60 to 70 Earth radii.

Archimedes:
Really remarkable, Aristarchus.
But what puzzled me in reading your work was how you actually managed to find the angle between the line of the Earth's center connecting to the Moon and for the line that connects the Moon with the Sun. Finding this angle then allowed you calculate the distance of the Sun from the Earth in terms of the distance between the Earth and the Moon.
He draws appropriate figures in the sand to illustrate this.

Aristarchus:
The method occurred to me when one evening I was looking at the First Quarter Moon. The line of the shadow seemed to be perfectly vertical, as seen from the Earth.
He holds up his plumb bob against the Moon.

Unfortunately, tonight we don't have a Quarter Moon, we have a full Moon. For that we would have to wait for another 13 or 14 days. But you can imagine the shadow covering one half of the Moon's surface,

Archimedes:
I see. … you can then assume that the angle that the Earth makes with the Sun is exactly a quarter circle.
He shows this on the drawing in the sand.

Aristarchus:
But how would you measure the small angle the Sun makes?

He points to the Moon and then toward the West were the Sun would have set earlier.

But this is too difficult to imagine. Let me see.
He now asks two persons, one man and the other a woman, to represent the Sun and the Moon to illustrate the smallness of the angle.

Archimedes:
Clearly, you cannot measure the angle directly. Then how did you determine this angle?

Aristarchus:
Now, that was a challenge.

He points to the full Moon.

Luckily, the solution turns out to be simple. Actually, you have to determine the large angle between the line connecting the Earth with the Moon and the line connecting the Moon with the Sun , instead of trying to measure the small angle directly.
He stops for a moment, shows this on the sketch made in the sand.

It then occurred to me that, if the time it takes from First Quarter to the Last Quarter are identical, then the Sun must be either at infinity or at a very long distance away.

Archimedes:
The idea of an immeasurable, or an infinite does not appeal to me as a mathematician.

Aristarchus:
Yes, I can understand that.
But ---I found that these times were different by about 1 day. Since I knew that a lunar period is 27 and a ½ days, and assuming that the motion of the Moon around the Earth is constant and circular, I estimated the angle the Earth and the Moon makes to be about 87 degrees.
Let me see. How can I make this clear?
Aristarchus again asks the two persons originally involved to represent the Moon and the Earth. The "Moon" rotates around the "Earth", from the position of First Quarter to the position of the Second Quarter, and then from this position back to the First Quarter. The times are compared (by counting "days" as the persons slowly "rotate") and found to be different by one day, that is 13 to 14.

Archimedes:
Very ingenious, my dear Aristarchus.
And then using geometry you calculated the distance to the Sun to be more than 18 but less than 20 times the distance from the Earth to the Moon.

Aristarchus:
Exactly.
He now explains the argument and the geometry. He draws a circle about ½ m in diameter and a much smaller one ,connected by a line about 3 m away. He then asks someone in the group to be the Sun, and has him/her move about 50 m away. He draws a line to the Sun and another line back to the Moon.

Archimedes:
This is very convincing, Aristarchus.
There is a pause as they look at the beach and the person representing the Sun about 50 m away.

Aristarchus:
But in the Sand Reckoner you put the upper limit of the distance to the Sun to 30 times the Earth diameter.

Archimedes:
Yes. As I mentioned earlier, I wanted to find the biggest number imaginable: that is, the number of grains of sand in the Universe, even larger than the one you computed.

Aristarchus:
Interesting.
He laughs and then turns to Archimedes.

You then invented a system of counting that allows one to count, in effect, how many poppy seeds there are in the Universe.

Archimedes:
Indeed.
He looks at Archimedes and says, very seriously:

We are engaged in the serious work of understanding the mechanics of motion of the planets and the stars and finding out the size of the Universe, but we must always retain a sense of humor, my dear Aristarchus!

Aristarchus:
I agree.
But still, how many poppy seeds, or grains of sand the size of poppy seeds, can you put into the Universe, Archimedes?

Archimedes:
Well, the number of grains of sand of the size of a poppy seed would be about a myriad myriad units of the eighth order of numbers.
Large but not infinite.
Archimedes now "steps out of character", looks at the group and says:

This is equal to about 10 taken to the power of 64, or 1 with 64 zeros behind it.
He stops for a moment to allow the audience to think about this.

Actually, if you calculate the number of atoms in a grain of sand, you get about 10 to the power of 14 "atoms". That means that Archimedes' universe would contain, 10 to the power of 78 "grains of sand, now called atoms.

Again, he stops for a moment to allow the audience to think about this.

What is astonishing is that in today's physics and astronomy texts you find that, indeed, the order of elementary particles in our universe is about 10 to the power of 78!

An interesting coincidence you must admit!
He turns back and faces Aristarchus.

Aristarchus:
I did try to follow your fascinating argument of how you devised this wonderful system of numbers when I studied your Sand Reckoner, but I was more interested in your arguments against my “wild idea” or hypothesis that the Sun and the stars are fixed and that the Earth and the planets revolve in circles around the Sun.

Archimedes:
In the Sand Reckoner I showed that in your Sun-centered planetary system the fixed stars would have to be at an immeasurable distance away. Let me show you why that is so.
He draws a small circle to represent the Sun, a much smaller one, about 3 meters away from the Sun, understood to be revolving around the Sun. He draws the 3m circle.

Now, imagine a star to be fixed at a distance so far that you could just measure, say 1 degree of parallax in six months time. That is, using the diameter of the Earth’s rotation as a baseline for calculating the distance to a star that makes a very small angle of say ½ degree, the same as the angle subtended by the Sun, as seen from the Earth.
He shows what he means in the sand.

How far would that star be from the Earth?
Let me demonstrate this.
He asks someone in the group to be the nearest star. This person then is asked to walk along the beach.

Aristarchus:
He smiles and nods.

Actually, my dear Archimedes, I did work this out some time ago..
The answer is about 200 times the distance between the Earth and the Sun.
He now shouts to the person who is walking away:

Sir, Madam, you will need to go out about 10000 paces!
The person stops and returns.

Archimedes:
Why would the sphere of the fixed stars be so far away?

Aristarchus:
Remember, that many astronomers and mathematicians in the past would not have believed that the distance to the Moon is about 70 times the radius of the Earth, and – certainly not that the distance to the Sun is 20 times the distance to the Moon!
He looks at Archimedes, who seems to be deep in thought.
He continues in a loud voice.

If all this is now is seen as plausible, why can we not accept that the distance to the stars is at least 200 times the distance between the Earth and the Sun? And perhaps there are many spheres of stars, proportionally further away.

Archimedes:
My friend, you are letting your imagination run wild!

Aristarchus:
I wander what Pythagoras of Samos would say to all this?

Archimedes:
I don't know. But I think he would be impressed by the power of geometry and mathematics in measuring the size of the Universe.

Aristarchus:
And Heraclitus of Ephesus?

Archimedes:
He would say: "I told you so".
They both laugh.

Aristarchus:
Puts his arm around the shoulder of Archimedes and smiles.

Come now, my young friend, let us go to my comfortable home, drink a glass of the fine wine made here in Samos and discuss the geometry and construction of the Tunnel of Eupalinos.
They begin to walk together.

Archimedes:
A good idea, Aristarchus.
Aristarchus stops and seems to remember something.

Aristarchus:
You must tell me about Eratosthenes in Alexandria and his idea of how to measure the size of the Earth. If we could do that, we would have a basis for all our astronomical distance measurements.

Archimedes:
Yes, I will.
He looks at Archimedes with a serious expression on his face.

Perhaps, our contrary ideas about the structure of the universe will meet at the end of the next conversation, much like the tunnel of Eupalinos, dug from opposite ends –almost – met in the middle.

They laugh heartily and slowly walk away. Archimedes stops and runs back to pick up his stick he left lying behind in the sand. He erases all the circles drawn in the sand.

We hear Greek music played from a distance.

END