Journey to Mars: the physics of travelling to the red planet

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Abstract

Mars has fascinated mankind since antiquity. The retrograde motion of the red planet provided the impetus for the Earth-centred solar system of Ptolemy, and 1500 years later, for the Sun-centred solar system of Copernicus. Kepler's laws of planetary motion were the result of his all-out 'war on Mars' that lasted for about 18 years. Fascination for Mars reappeared in the beginning of the last century with the astronomer Percival Lowell's well publicized claim that intelligent life was responsible for the 'canals' that were sighted with a new powerful telescope. We are seeing a resurgence of this interest in the wake of many successful attempts to land on Mars in the last 30 years to study the surface and the atmosphere of the planet. Indeed, the Canadian Space Agency (CSA) is now cooperating with NASA in the quest for a full scale scientific assault on the red planet. In response to this new interest, we wrote an interactive computer program (ICP), illustrating the physics of planetary motion, that we have used successfully in lecture-demonstrations and with students in classrooms. The main part of this article describes two missions to Mars, and a third one that illustrates the capabilities of the ICP.

Introduction

Studying the motion of the 'wanderers' in the sky, especially the retrograde meandering of Mars, provided the impetus for challenging the astronomers in ancient Greece to find a model for the solar system. Plato's injunction 'By the assumption of what uniform and ordered motions can the apparent motions of the planets be accounted for?', eventually resulted in the Ptolemaic Earth-centred solar system, 500 years later.

Copernicus, some 1400 years after Ptolemy, studied the motion of Mars carefully and argued for a sun-centred solar system where all planets moved in circular orbits. Using this model, the retrograde motion of Mars and the phases of Venus were easily accounted for. Kepler, in an attempt to improve on Copernicus' model of circular motion, showed in his 18 year 'war on Mars' that the planet's path was elliptical and not circular. Newton then used Kepler's laws of planetary motion to corroborate his inverse square law of gravitational attraction.

Mars again captured the attention of astronomers around 1900, when the American astronomer Percival Lowell, using a new 21 inch (53 cm) telescope in Flagstaff, Arizona, concluded that there were 'man-made' canals on Mars. In 1908 he published a book entitled *Mars as the Abode of Life*, which caused a sensation in America and in Europe. Today we know that there are no such canals on Mars (Griffiths 2003).

Since 1975, starting with the Viking missions,

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NASA has explored the planet, providing scientific information for almost six years. Since then there have been many visits to the planet; a brief search on the internet will attest to that.

A recent issue of the accessible British magazine *New Scientist* contained a special report 'The Moon and Mars' that featured the article 'Destination Mars' (Mullins 2004). This special report was published in response to the new interest in space exploration that the American president George Bush created in 2003 with his support of a program aimed at a human landing on Mars. He did not set a date, probably because George Bush senior set an unrealistic one (2020) some years ago.

Reading this article reminded us of a previous special issue of *Scientific American* that we used to write an interactive computer program (ICP) for our students for studying elementary celestial mechanics. See www.hsse.uwinnipeg.ca.

The March 2000 issue of *Scientific American* presented a special report entitled 'Sending Astronauts to Mars'. The articles were written by experts in the field of space travel and were sufficiently detailed that it is possible to use the description and the data presented to map out a 'large context problem' for high school and first-year college students (Stinner 2000). This special issue provides a chance to develop an interesting and realistic context with a central 'big idea' that attracts students' interest and excites their imagination.

In one of the articles, space scientist Fred Singer briefly describes six possible scenarios to travel to Mars, and in another, Robert Zubrin, president of the Mars Society, gives a sufficiently detailed description of the trajectories involved to generate interesting and realistic problems for an elementary physics class.

Finally, a recent report from Toronto's *Globe and Mail* (January 15, 2004), taken from the internet, states that:

> Space researchers and enthusiasts say Canada's expertise and ambitions could dovetail nicely with US president George W Bush's plans for using a lunar base as a launch pad for a manned Mars mission. The Canadian Space Agency's (CSA) Marc Garneau, who on Tuesday said that the agency has its own plans for a Mars mission, ... "In Garneau's eyes and the CSA's eyes, Mars is a big target right now", said Matt Bamsey, president of the Mars Society of Canada".

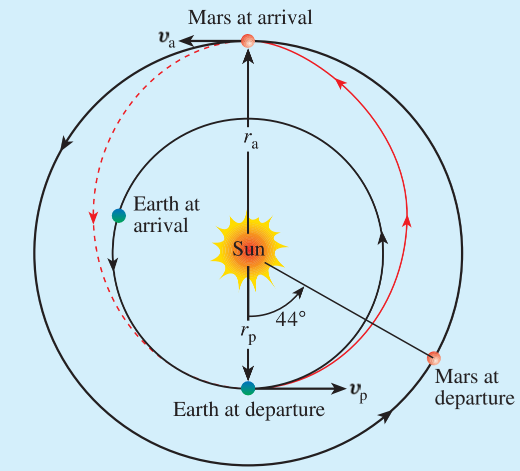


Figure 1. A Hohmann orbit transfer (HOT) trajectory.

We are not attempting to describe the physics of launching a mission from the Moon, but from the Earth directly. What we offer here is a text that should be read with the ICP activated. We have made the ICP available on our website: www.hsse.uwinnipeg.ca.

We will describe two missions to Mars, using the data given in Zubrin's article as a guide for developing and solving problems that can be discussed in an elementary physics classroom. In Mission 1 we plan a trip to Mars based on a Hohmann orbit transfer (HOT) trajectory, which allows for a long stay on Mars, using minimum energy expenditure. This special transfer is discussed below (see figure 1).

In Mission 2 we discuss landing on Mars for only a brief time, then returning to Earth on a different trajectory, thus cutting the total time of the trip by a considerable amount. For our study the trajectories for a 30 day stay will be described. This type of scenario was rejected by the *Scientific American* author because of the high energy consumption involved and the short time of actual stay on Mars that such a trip would allow. However, the 'hitchhiking' space trip is especially interesting to us for pedagogical reasons, because it allows for the development of an exciting interactive program that responds well to students' 'what if' questions. Moreover, this exercise will liberate us from the constraint of the HOT trajectory, as we shall see.

The ICP is described in detail on the website above. *The authors assume that the reader will run the program from the website before reading the article and then also use it during the reading of the article.*

We have added some details for a third trip that allows staying on Mars for 100 days, using the ICP, and added some relatively simple calculations to describe it. We present this trip as an example of a study that students can make using our interactive tool.

Preliminary calculations

Before discussing the two missions to go to Mars we require a few preliminary calculations in preparation for the planning of the trajectories. Most of these are straightforward and can be found in introductory physics textbooks.

What is the escape velocity from Earth and from Mars?

The escape velocity from a planet is obtained by equating the gravitational potential energy from infinity to the surface to the kinetic energy required to overcome that potential energy:

$$\tfrac{1}{2}mv_{\mathrm{esc}}^2 = GmM_{\mathrm{planet}}/R_{\mathrm{planet}}.$$

Substituting values for the Earth and Mars, we obtain 11.2 $\mathrm{km\,s^{-1}}$ and 5.1 $\mathrm{km\,s^{-1}}$ respectively. These values will be important for our calculations.

What is the 'gravitational sphere of influence' of Earth and of Mars?

When planning our trip to Mars we will need to know how far we must be before we can ignore the effect of the gravitational attraction of the planet (or how close before we must consider the gravitational effect of the planet).

The generally accepted distance for the Earth's 'sphere of influence' is about 9.2×10^8 m, or more than twice the distance between the Earth and the Moon. The sphere of influence of Mars is about 5.7×10^8 m, or about 1.4 times the distance between the Earth and the Moon. To calculate these values astronomers generally use the suggestion made by Laplace about 200 years ago:

$$R_{\mathrm{S}} = D(M_{\mathrm{planet}}/M_{\mathrm{Sun}})^{2/5}$$

where D is the distance between a planet and the Sun. At this distance from the Earth, or from Mars, the gravitational attraction of the Earth or Mars should be negligible in comparison with that of the

Box 1. Orbital velocity and period.

(1) To find the **velocity** of a body orbiting about the Sun: Use the *vis-viva* equation, which is based on the total energy of an orbiting body:

$$E_{\mathrm{total}} = \tfrac{1}{2}mv^2 - GM_{\mathrm{S}}m/r$$

This total energy can be shown to be equal to $-GM_{\mathrm{S}}m/2a$. Therefore,

$$v_{\mathrm{r}} = [GM_{\mathrm{S}}(2/r - 1/a)]^{1/2}$$

where G is the universal gravitational constant ($6.67 \times 10^{-11}\ \mathrm{m^3\,s^{-2}\,kg^{-1}}$ and M_{S} is the mass of the Sun (2.00×10^{30} kg). This very important and universal equation is generally known among astronomers as the *vis-viva* equation. See figure 2.

(2) It is more convenient to write the *vis-viva* equation in the form

$$v_{\mathrm{r}} = 29.8 \times (2/r - 1/a)^{1/2} \quad (\mathrm{km\,s^{-1}})$$

where the 'average' velocity of the Earth is 29.8 $\mathrm{km\,s^{-1}}$ and a and r are given in astronomical units (AU). The distance to the Sun from the Earth is 1 AU (1.50×10^{11} m).

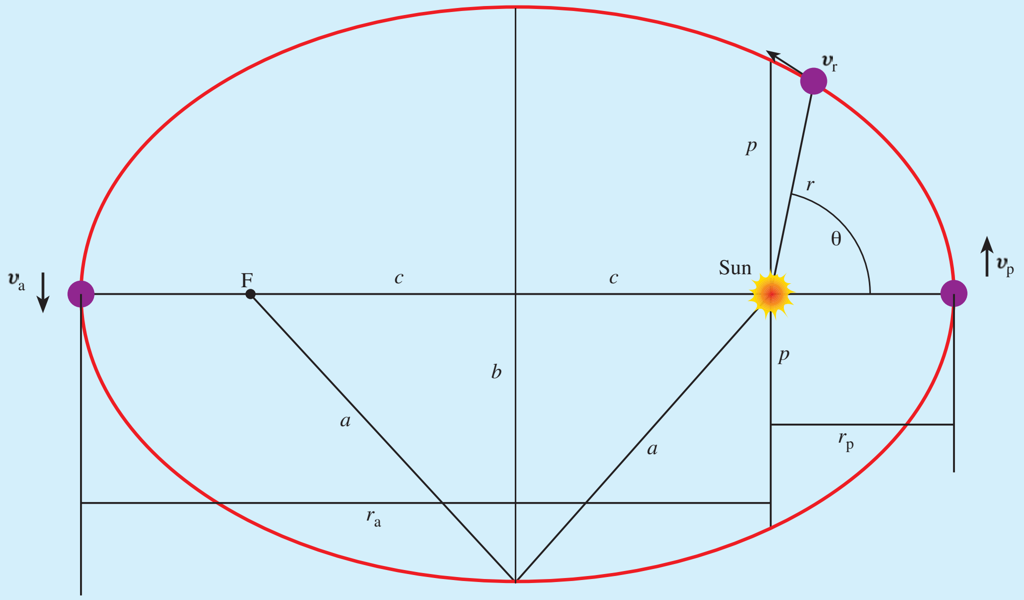
(3) The **period** of a body moving around the Sun is given by Kepler's third law:

$$P = P_{\mathrm{E}}a^{3/2}$$

where P_{E} is the period of the Earth and a is the semimajor axis. We then write

$$P = 365a^{3/2}$$

where we will take 365 days as the period of the Earth.



F = Focal point

$a = (r_p + r_a)/2$	a = semimajor axis
$e = c/a = (a - r_p)/a$	r_p = perihelion distance (closest to Sun)
$b^2 + c^2 = a^2$	r_a = aphelion distance (farthest from Sun)
$r = p/(1 - e\cos\theta)$	e = eccentricity of the orbit
$p = a(1 - e^2)$	θ = angle, defined
$\boldsymbol{v}_r = 29.8\ (2/r - 1/a)^{1/2}$	$\boldsymbol{v}_r$ = the velocity of a body revolving around the Sun at a given distance r
	p = half the latus rectum

Figure 2. The ellipse and its application to orbits.

Sun. We can easily check these values by using the inverse square law. See Box 1.

These values will become important when considering the motion of the spacecraft (SC) leaving or approaching the Earth or Mars. Certain simplifying assumptions will have to be made when calculating the motion here, because the combined effect of the attraction of a planet and the Sun on the spacecraft is a very complicated problem and cannot be handled using elementary mathematics.

Data and assumptions

The Earth has an almost circular orbit but Mars's orbit is more eccentric. For our calculations, however, we will assume that both orbits are circular, with radii equal to their respective semimajor axes, 1.00 AU and 1.52 AU, and also assume that their orbital velocities are constant (29.8 km s^{-1} for the Earth and 24.1 km s^{-1} for Mars). (One AU, or Astronomical Unit, is the average distance between the Earth and the Sun, or 1.50×10^{11} m, as shown in figure 2.) Moreover, since the inclination of the orbit of Mars is only about 1.8° to the orbit of the Earth we can safely consider the two orbits to be coplanar.

The periods of the Earth and Mars will be taken as 365 d and 687 d, respectively. The period of Mars is calculated using Kepler's third law in the form $P = 365a^{3/2}$, where P is the period in days and a is the semimajor axis, given in AU. The semimajor axis of the Earth is then taken as 1.00 AU.

The following will be assumed for our calculation:

- When calculating the perigee velocity (the velocity required when the SC leaves the Earth) for the HOT trajectory, we assume that the SC is beyond the 'sphere of influence' of the Earth. That is, the calculation is made *as if the Earth did not exist*.

- When calculating the apogee velocity required to connect with the orbit of Mars, it is assumed that the SC will be outside the 'sphere of influence' of Mars. That is, the calculation is made *as if Mars did not exist*.
- To establish the energy budget for each trip, we use the Δv measure in km s^{-1}, following the practice of NASA. We have to remember that when adding Δv's, the absolute value must be taken.

Mission 1: Using a 'HOT line' to Mars

In 1925 the German engineer-astronomer Walter Hohmann showed that the trajectory requiring the minimum energy to go to Mars would be the one shown in figure 1. (We are using the acronym HOT to indicate a 'Hohmann Orbit Transfer' trajectory.) Most trips to Mars so far have used the HOT trajectory method.

The HOT trajectory between two circular (or near-circular) orbits is one of the most useful manoeuvers available to satellite operators. It represents a convenient method of establishing a satellite in high altitude orbit, such as a geosynchronous orbit. For example, we could first position a satellite in LEO (low Earth orbit), and then transfer to a higher circular orbit by means of an elliptical transfer orbit that is just tangential to both of the circular orbits. In addition, transfer orbits of this type can also be used to move from a lower solar orbit to a higher solar orbit, e.g. from the Earth's orbit to that of Mars.

The HOT trajectory requires the lowest energy. This can be shown by calculating the energy requirement of trajectories that would meet Mars at progressively later times. We have done this for the trajectory that connects at $\theta = 90°$ (see figure 3). This position is ideal for a straightforward solution using the equations given in figure 2. We find that the SC would need to follow a trajectory with a semimajor axis a of 2.08 AU and an eccentricity of 0.52. The perihelion velocity required is 36.7 km s^{-1} and the velocity of the SC arriving in the vicinity of Mars is 27.2 km s^{-1}. Granted, the time of transit would be only 88 d, as compared with the 228 days needed for Mission 2. However, the Δv for this trajectory is clearly prohibitive: 18.1 km s^{-1} to place the SC into the trajectory and an additional -8.2 km s^{-1} to land on Mars, for a total of 26.3 km s^{-1}.

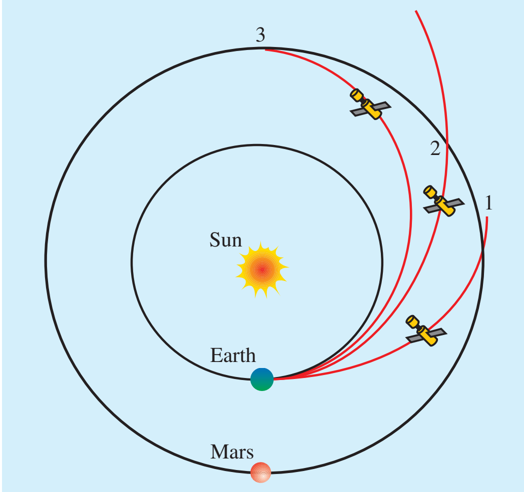
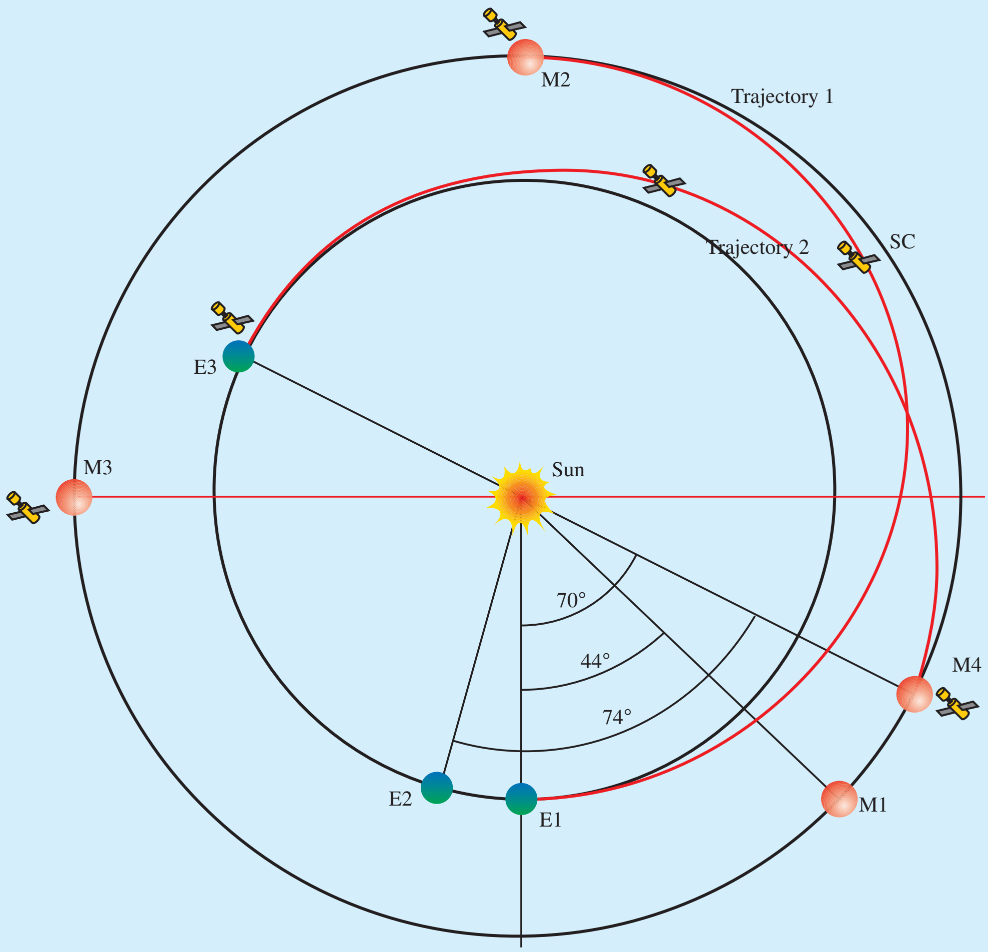


Figure 3. Trajectories to connect with Mars. Trajectory 1: short time, high $\Delta \boldsymbol{v}$. Trajectory 2: longer time, still a high $\Delta \boldsymbol{v}$. Trajectory 3: HOT trajectory: longest time but least $\Delta \boldsymbol{v}$.

We will see that the Δv for the HOT trajectory to get to Mars is only about 16.7 km s^{-1}. Since energy is proportional to the square of the speed, the short flight would require about 2.5 times the energy. (We are also neglecting the fact that the SC approaches Mars at an angle of about 30° to the orbit of the planet, which increases the Δv value considerably.) The HOT trajectory, therefore, is the longest but it is the least energy-demanding choice.

Calculating the ellipse and the velocities for our HOT line to Mars

We begin by calculating the period of the ellipse for the Mission 1 trajectory where r_a is 1.52 AU and r_p is 1.00 AU, with a semimajor axis of 1.26 AU. Using Kepler's third law $P = 365a^{3/2}$, we find that the period of the HOT trajectory is about 516 days and therefore the time of flight to Mars will be 516/2, or 258 days. Next, we use the *vis-viva* equation, $v_r = [GM_S(2/r - 1/a)]^{1/2}$, to find that v_p, the velocity required to leave the Earth (after escaping the influence of Earth's gravity), would be 32.7 km s^{-1} (relative to the Sun). Finally, we calculate the velocity v_a that the SC would have when approaching Mars to be 21.5 km s^{-1} (see figure 2). The Δv for escaping the Earth and being



Trajectories 1 and 2

$a = 1.26$ AU
$r_p = 1.00$ AU
$r_d = 1.52$ AU
$e = 0.21$
$P = 517$ d

SC leaves Earth on trajectory 1 when Mars is ahead by 44° (E1) and arrives at Mars 258 days later (M2), March 28, 2006.
SC and crew are on Mars (M2–M3–M4) for 458 days.
SC leaves Mars on trajectory 2 when Mars is about 74° ahead of Earth (E2).
SC and crew in transit for another 258 days.
SC and crew arrive on Earth (E3).
The total time for the trip is 974 days (2.7 years), returning on about March 13, 2008.

Figure 4. Travel schedule for Mission 1. Possible departure date from Earth: July 13, 2005.

injected into the HOT orbit is $(11.2 + 2.9)$ km s^{-1}, or 14.1 km s^{-1}.

One more adjustment must be made, however. Since it takes about 18 hours for the SC to reach the radius of gravitational influence of the Earth, at a distance of about 9×10^8 m, the launching should take place $1\frac{1}{2}$ days *before* the calculated date of departure. The SC can now be considered as moving only under the gravitational influence of the Sun, until it approaches Mars at a distance of about 6×10^8 m from Mars, when the gravitational influence of Mars becomes dominant.

When the SC approaches Mars, about 256 days later, it will be pulled towards the planet. The orbital velocity of Mars at 24.1 km s^{-1} is larger than the approach velocity of about 21.5 km s^{-1} of the SC. The SC should, therefore, arrive a little *ahead* of Mars and allow the planet to 'catch up' with it. The Δv now would be -2.6 km s^{-1}, since the escape velocity from Mars is 5.1 km s^{-1}. The

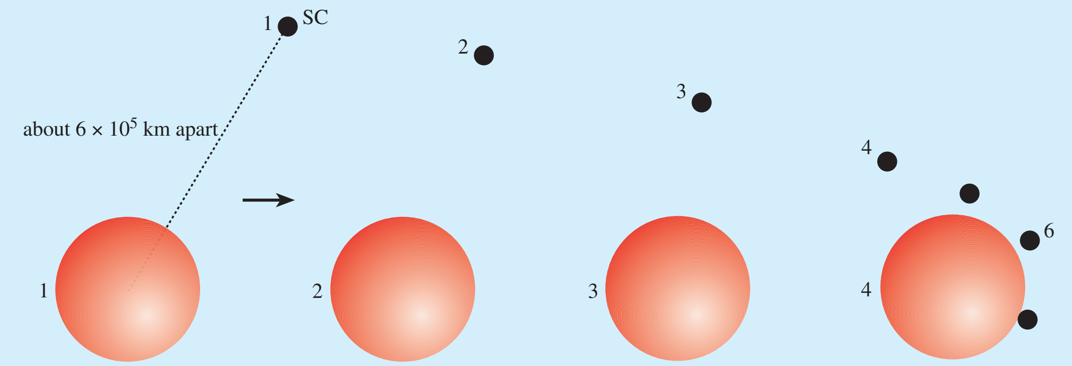


Figure 5. At point 1 the SC enters the gravitational influence of Mars. The velocity of the SC here is about 21.5 km s^{-1} (relative to the Sun). Mars is moving at a constant velocity of 24.1 km s^{-1}. Mars therefore overtakes the SC and gravitational attraction now pulls in the SC. The speed of the SC increases rapidly and the SC is pulled into the planet. If retroactive rockets were not applied the SC would either crash into Mars or swing around the planet at about 5.1 km s^{-1}, escaping into space. At point 6 the speed of the SC is about 3.5 km s^{-1} at a height of about 100 km and it could be placed in a circular orbit. The SC then lands on the surface of Mars.

retroactive rockets will therefore be engaged to achieve a Δv of (2.5−5.1) km s^{-1}, or −2.6 km s^{-1}. The total Δv for the trip then is (14.1+2.6) km s^{-1}, or 16.7 km s^{-1}. See figure 4.

Using the ICP, it is easy to show that the SC will have to stay on Mars for 458 days, because it takes that long before the Earth and Mars are again in a position to initiate a second HOT manoeuver to return to Earth. This time, however, Mars must be about 74° ahead of Earth when the launch takes place, allowing Earth to 'catch up' with the SC in about 258 days (the HOT trajectory period divided by 2). Again, this can be checked using the ICP. See figure 4. The return flight involves escaping the gravity of Mars. This time the SC, after escaping the gravity of Mars, has to slow down to an orbital velocity of 21.5 km s^{-1}. To achieve this, the SC escapes the gravitational pull of Mars by leaving in the *opposite direction* to the orbital motion of Mars. First, the SC must leave the gravitational sphere of influence of Mars and then apply retroactive rockets to slow the orbital velocity (relative to the Sun) from 24.1 to 21.5 km s^{-1}. This will require a Δv of 5.1 km s^{-1} and −2.6 km s^{-1}, for a total Δv of 7.7 km s^{-1}.

As before, the SC will be in the vicinity of the Earth in about 256 days, approaching the Earth with a velocity of 32.7 km s^{-1}, or about 2.9 km s^{-1} faster than the orbital velocity of the Earth. The Earth will pull in the SC, just like Mars did, and if no retroactive rockets are used the SC would fall into the Earth with a velocity of about $(2.9^2+11.2^2)^{1/2}$ km s^{-1}, or 11.6 km s^{-1} (see our website). This is a little larger than the escape velocity of 11.2 km s^{-1}. So our Δv is about 11.6 km s^{-1}. (Of course, NASA might decide to use the atmosphere to slow down the SC.)

According to the ICP, we find that if we left on about July 13, 2005, it is possible to connect with Mars, as planned. We would return to Earth 978 days later, on about March 13, 2008, a travel time of about 2.7 years! The reader is encouraged to check these dates using the ICP. Adjust the Earth to Mars section (Go Date) to July 13, 2005. Adjust the perihelion radius to 1.0 (sometimes 0.99 is better), then under 'Mars to Earth' adjust the 'Go Date' to June 29, 2007. Study the configuration before starting the program. You can check all the dates and the number of days that pass by stopping the action at any time.

The energy budget for Mission 1

To travel from Earth directly on the HOT trajectory will require a Δv of about 14.1 km s^{-1}. The SC arrives at the orbit of Mars with a velocity of 21.5 km s^{-1}, or about 2.6 km s^{-1} slower than the orbital velocity of the planet. Therefore the SC must arrive ahead of Mars and pass inside the 'sphere of gravitational influence' so that the SC is pulled in (see figure 5). The escape velocity of Mars is 5.1 km s^{-1}, therefore the Δv needed for landing the

Table 1. The energy budget and time for the missions.

Trip	Type	Time to reach Mars	Total time away	Time on Mars	Δv (total) ($km\,s^{-1}$)	Energy per unit mass ($J\,kg^{-1}$)
Mission 1	HOT trajectory both ways	258 d (0.70 y)	978 d (2.7 y)	460 d (1.3 y)	$14.1 + 7.7 + 11.6 = 33.4$	2.0×10^8
Mission 2	Modified HOT trajectory	226 d (0.62 y)	490 d (1.34 y)	30 d	$14.1 + 2.6 + 7.5 + 14.2 = 38.4$	2.3×10^8

SC is $(2.6 - 5.1)$ $km\,s^{-1}$, or -2.6 $km\,s^{-1}$. This is a very small Δv requirement.

Returning to Earth the SC needs to initially move in the *opposite* direction to the motion of Mars; first overcoming the gravity of the planet (5.1 $km\,s^{-1}$) and then 'slowing down' to the apogee velocity of 21.5 $km\,s^{-1}$ from the orbital velocity of 24.1 $km\,s^{-1}$, again for a total Δv of 7.7 $km\,s^{-1}$. As we have shown on the preceding page, the total Δv for the whole trip then is (14.1 + 7.7 + 11.6) $km\,s^{-1}$, or 33.4 $km\,s^{-1}$. We can express the total energy as the sum of squares of the individual Δv's in $m\,s^{-1}$, and dividing by 2 we have an energy consumption of about 2.0×10^8 $J\,kg^{-1}$. See table 1.

Mission 2: A brief visit to Mars

In this mission the SC travels to Mars on a modified HOT trajectory, lands on the planet for 30 days only, and then returns to Earth by way of a trajectory that has a high eccentricity. For the sake of simplicity we have the SC approach Mars along the regular HOT trajectory but intersecting the orbit of Mars 30 days earlier. The SC lands on Mars, stays for 30 days and then leaves at the aphelion point.

On the return trip the SC may cross the orbit of Venus and could even be inside the orbit of Mercury for a brief time. Use the ICP to design different times for staying on the planet and come back on various trajectories. For example, the interactive program can be used to plan journeys for a flyby of Venus and/or Mercury.

This scenario was rejected by NASA because it would be too energy-intensive for just a brief stay on the planet. However, the scenario, unlike Mission 1 and the trip described in the next section, provides a good context for discussing a number of interesting and challenging situations. It also lends itself to having students plan their own journey to Mars.

Mission 2 to Mars will be almost identical to a full HOT trajectory, with the exception of the initial separation of Earth and Mars. Mars will have to be about 46° ahead of the Earth, rather than 44° in order to connect with the planet within the region of gravitational influence about 30 days before apogee. See figure 6.

The velocity of the SC at perigee, as for Mission 1, must be 32.7 $km\,s^{-1}$, so that Δv is 2.9 $km\,s^{-1}$ + 11.2 $km\,s^{-1}$, or 14.1 $km\,s^{-1}$, as before. We assume that the velocity upon arrival in the region of the gravitational influence of Mars is also about 21.5 $km\,s^{-1}$. The period of the trajectory is again 516 days and therefore the time of travel $(516/2 - 30)$ days, or about 228 days. The Δv for matching the orbital velocity of Mars will again be about 2.5 $km\,s^{-1}$.

The return trip to Earth

For Mission 2 the plan is to stay on Mars for 30 days and return on a trajectory that begins at the apogee point. It will cross the orbit of Venus and possibly the orbit of Mercury. The reader should find this return trajectory before reading any further.

After much trial and error we found that the trajectory that has a perigee of 0.42 AU guarantees connecting with the Earth in about 235 days. Note that on the return trip the SC would come very close to the orbit of Mercury. The SC must leave Mars in the opposite direction to its motion around the Sun, overcome the gravity of the planet and then slow down to 21.7 $km\,s^{-1}$. This amounts to a Δv of 7.2 $km\,s^{-1}$. The total time then for the whole trip is 490 days, or 1.34 years. This is much shorter than the 2.7 years it takes for Mission 1.

The energy budget for Mission 2

Escaping Earth will require the same Δv as for Mission 2, namely 14.1 $km\,s^{-1}$. Landing on Mars

Box 2. Trajectories for Mission 2.

Trajectory 1 is the same as the HOT trajectory we calculated for Mission 1, except that we must leave Earth when Mars is 46° ahead of the Earth in order to approach the planet about 30 days before arriving at the aphelion point. The trip will last about 226 days, and 30 days later the SC returns on trajectory 2.

Trajectory 2. This is a more complicated orbit to calculate than the simple HOT trajectory. The time of transit along an ellipse can be calculated from Kepler's second law and is equal to the area under the portion of the ellipse determined by the angle θ and divided by the total area of the ellipse. The geometric solution to this problem was worked out by Kepler, but it took the power of the calculus to provide an analytical solution. The area of an ellipse is simply πab.

Therefore, the transit time is

$$T_{\text{transit}} = \frac{\text{Area under the portion of the ellipse considered}}{\pi ab}.$$

A fairly straightforward geometric argument leads to Kepler's equation:

$$T_{\text{transit}} = 365a^{3/2}(E - e\sin\theta)/2\pi$$

where

$$E = \cos^{-1}\left(\frac{e + \cos\theta}{1 + e\cos\theta}\right).$$

(Care must be taken, however, to express E in radians!)

There are many choices we can make in deciding Trajectory 2. Our perihelion distance, r_p, of course, is fixed at 1.52 AU. The perihelion distance, r_p, is constrained by how close we dare to come to the Sun. If we chose to go deep inside the orbit of Mercury, say for $r_a = 0.30$ AU, the radiation energy from the sun would be $(1/0.3)^2$ times that on Earth, or about 11 times greater. We found that a trajectory that has a perigee of 0.42 AU (very close to the orbit of Mercury) connects with the Earth in about 235 days. See our website for details for this calculation.

will involve, as for Mission 1, a Δv of only 2.6 km s^{-1}.

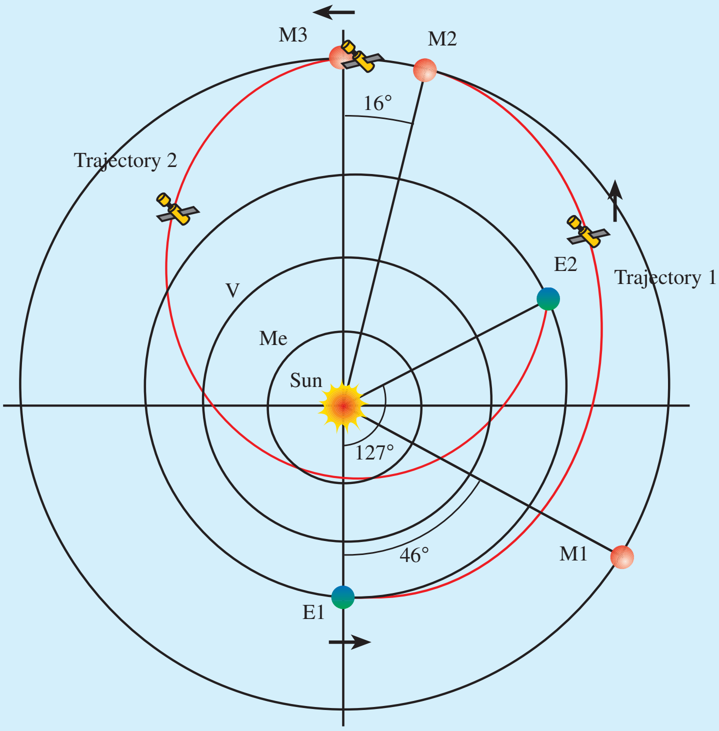
Coming back, first escaping Mars (in the direction opposite to its motion) takes 5.1 km s^{-1}, and then slowing down to 21.3 km s^{-1} takes -2.4 km s^{-1}, a total of 7.5 km s^{-1}.

When arriving in the vicinity of the Earth, first slowing down to the orbital velocity of Earth (29.8 − 32.8), or -3.0 km s^{-1}, and then overcoming the gravitational attraction of the Earth adds another Δv of -11.2 km s^{-1}, for a total of -14.2 km s^{-1}. The Δv for the whole trip then is (14.1 + 2.6 + 7.5 + 14.2) km s^{-1} = 38.4 km s^{-1}. The energy consumption then is $\frac{1}{2}(14.1^2 + 2.6^2 + 7.5^2 + 14.2^2) \times 10^6$ J kg^{-1}, or 2.3×10^8 J kg^{-1}.

Another trip using the ICP

Let us find out (by trial and error) what the trajectory must be if we leave on July 13, 2005 for Mars on a HOT trajectory, stay for 100 days and then come back on a trajectory that connects with the Earth. The time to reach Mars would be 258 days, as before.

It is easy to adjust the parameters of the return trajectory to find the suitable perihelion distance r_p so that the SC connects with the Earth. The SC lands on Mars 258 days later and stays on Mars for 100 days. The return trip then starts on July 6, 2006. After some trial and error, we find that if r_a, or the closest approach to the Sun on the return trip is about 0.26 AU (inside the orbit of Mercury!) the SC arrives back on Earth on about January



Trajectory 1

$r_p = 1.00$ AU
$r_a = 1.52$ AU
$a = 1.26$ AU
$P = 522$ d
$e = 0.21$

Trajectory 2

$r_p = 0.41$
$r_a = 1.52$ AU
$a = 2.97$ AU
$P = 349$ d
$e = 0.57$

The SC leaves Earth (E1) on Trajectory 1, when Mars is about 46° ahead.
226 days later it arrives at Mars and lands for 30 days.
The SC leaves on Trajectory 2 (M3), crosses the orbits of Venus and Mercury and arrives on Earth (E2), 234 days later on November 18, 2006. The total trip lasts 490 days (1.34 years).

Figure 6. Travel schedule for Mission 2. Possible departure date from Earth: July 13, 2005.

7, 2007, approximately 563 days, or 1.54 years later. The energy budget could now be worked out and compared with the two missions we have described.

Using the *vis-viva* equation we can calculate the velocity of the SC as it is moving past the Sun at the closest approach of 0.26 AU to be almost 60 $\mathrm{km\,s^{-1}}$. The radiation energy received per unit time per unit area from the Sun would be $(1/0.26)^2$, or about 15 times that which we receive at the top of our atmosphere! NASA would surely reject such a trip.

Looking at table 1, we note two important things about the energy consumption. First, the energy consumption of the two trips differs by only about 15%. Secondly, the energy required per unit mass (about 2×10^8 $\mathrm{J\,kg^{-1}}$) to go to Mars is enormous, when we realize that 1 kg of the explosive TNT is equivalent to an energy of 4.1×10^6 J. So more than 90% of the load must be fuel and the 'payload' is less than 10%.

Concluding remarks

Few high school and first-year university physics students understand the physics of travelling to a planet beyond isolated problems they solved in their textbooks. This paper in conjunction with our ICP may provide enough background and material for teachers of physics to enrich their presentation of the physics of Newtonian gravitational theory, Kepler's laws and their application to space travel.

We did not discuss the psychological and physiological problems that long travels in a confined space, and effectively in 'free fall', will present. A thorough reading of Sarah Simpson's 'Staying Sane in Space' (in the *Scientific American* issue mentioned) would provide enough background for a good discussion of these problems and may suggest further research. Surely, the physics of space travel as well as the technical problems of going to Mars can be solved; but whether or not we can ever solve the human aspects of such a journey is an open question. Whichever scenario NASA decides to choose, the time of travel will be more than one year. A more recent and easily accessible reference is that of Mullins (2004).

Aside from the already stated simplifications used in solving the problems (circular motion of the planets, and their coplanar positions) we also neglected to take into account the problem of the angle of approach when a trajectory intersects with the orbit of a planet (see Stinner 2000). For the Hohmann trajectory that problem clearly does not arise since it overlaps the orbit of the planet on contact. Finally, it should be mentioned that NASA may use the braking effect of the atmosphere of Mars and that of the Earth when spacecraft land, thus reducing the Δv factor by a significant amount for a journey.

A quick reference to table 1 allows a comparison of the two missions, based on energy requirements, time spent on Mars and the total time necessary for a return trip. This table could stimulate a lively discussion trying to balance the advantages and disadvantages of the two missions. Students can now propose their own missions and present the results in a table, similar to table 1.

The German engineer Walter Hohmann showed, in 1925, how to select a trajectory to a planet like Mars that would require the least energy. At that time Charles Lindbergh's crossing of the Atlantic by a plane was still in the future and space rockets were science fiction. Hohmann was a visionary, earning the respect and admiration of the young Wernher von Braun, who later used his calculations and technical suggestions to plan the first landing on the Moon. The Hohmann orbit transfer is used by NASA, along the lines discussed in this paper. The story of Walter Hohmann provides an engaging educational link between the space visionaries of the past and the present possibilities of modern space technology. Students should be given the opportunity to participate in this adventure.

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