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The story of force: from Aristotle to Einstein

Arthur Stinner

This article summarizes for the physics educator the conceptual development of the notion of force from Aristotle to Einstein, suggesting appropriate analogies, limiting case analyses, thought experiments and imagistic representations that can be used in high school physics.

Researchers have found clear parallels between students' intuitive conceptions in science (mechanics, electricity, heat) and historical prescientific conceptions (McCloskey 1983, Nersessian 1989). Although this finding suggests that it may be possible to have the learning process recapitulate the historical process, closer examination of the complex thinking involved in scientific discovery shows that setting such a goal is probably unreasonable (Nersessian 1989). A plausible case, however, can be made for a limited recapitulation of the historical process in domains, such as pre-Newtonian mechanics, that are experientially familiar to the students.

For pre-Newtonian physics the conceptual development depends on commonsense perceptions based on personal kinesthetic memory. On the other hand, post-Newtonian concepts are related to internalist notions such as thought experiments, which may be difficult to connect to ordinary experience. Moreover, it may be that physics teachers themselves have generally limited acquaintance with the ideas of Mach and Einstein. Teachers therefore tend to believe that the 'discovery argumentation' (Nersessian 1989, p 179) required for presenting these ideas would be too difficult for beginning physics students. It may, however, be possible to also achieve partial recapitulation of post-Newtonian ideas of force and motion with high school physics students.

The aim of this brief discussion then is twofold: to summarize for the physics educator the conceptual development of the notion of force from Aristotle to Einstein, and to suggest appropriate analogies, limiting case analyses, thought experiments and imagistic representations for partial recapitulation of the historical process of the concept of force. High school physics students are generally very interested in Aristotle's ideas about force and motion. They are also fascinated by the claims of modern physics, especially Einstein's ideas. But the ideas of Aristotle about force and motion are generally dismissed or trivialized by textbook writers, and those of Einstein are abstract and often made inaccessible to high school physics students.

This paper argues that a history-based exposure to the conceptual development of Newtonian mechanics is superior to a conventional textbook-centred approach, because it is contextual, shows the intellectual struggle involved in scientific thinking and relates better to students' knowledge and experience. The questions asked by post-Newtonian physicists about such assumptions as absolute space then make more sense to students, leaving open the door for a post-Newtonian discussion of force at an early stage.

Aristotle and the notion of force

A close look at Aristotle's physics reveals that he was a keen observer of nature, not only as a naturalist, but also as a physicist. Let us look at his understanding of the notion of force and motion in some detail. The science of the Greeks in general, because it is essentially high-grade thinking based on unaided observation, seems especially well suited for discussion prior to problem-solving activity of the textbook type.

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Aristotle looked at local motion as either *natural* or *violent*. Natural motion was seen by him as *celestial* motion (which is uniformly circular and perpetual) or as *terrestrial* motion (which is rectilinear, up and down and finite). All other motion was classified as violent (see figure 1).

Aristotle observed nature and reported what he saw. He saw that objects will come to rest when the force is removed. Thus a cart will come to rest when the horse stops pulling it. Moreover, if objects fall they move through a medium, such as air or water. Motion through a vacuum was considered impossible. In short, Aristotle saw a world in which there was *always* a resistance offered to the motion: this is the reality of motion. We can represent the role of force for an object moving through a medium by committing a slight anachronism as follows: velocity is directly proportional to force and inversely proportional to the resistance of the medium, or

$$V \propto \frac{F}{R}.$$

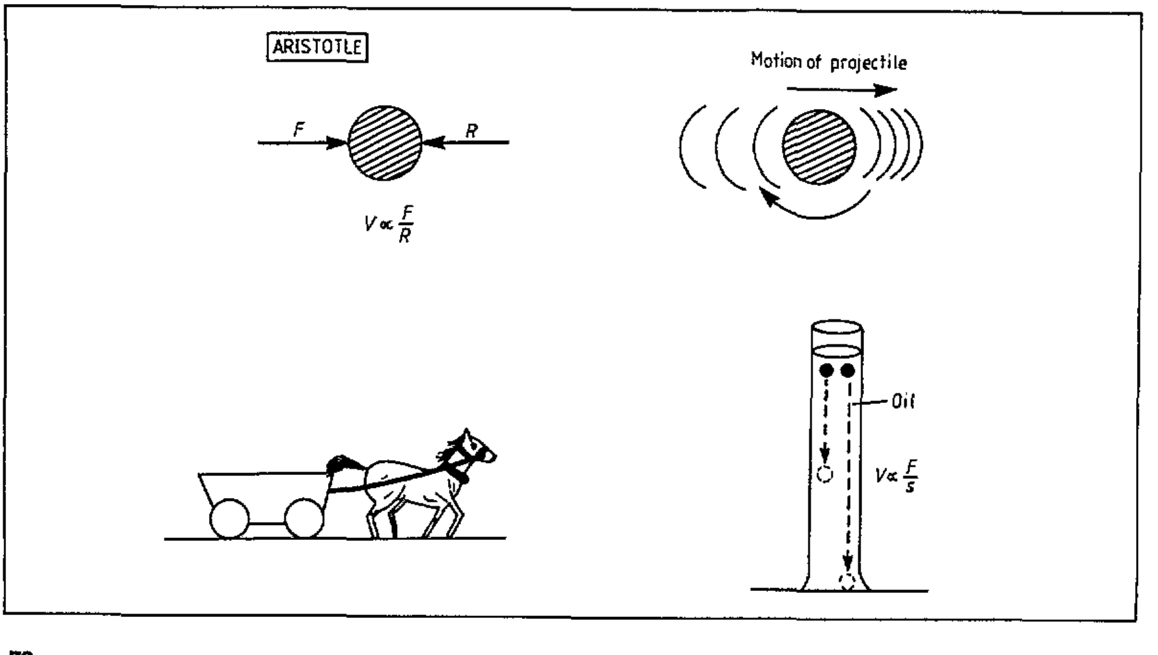
Aristotle argued that in free fall a given force (weight) produces a certain constant velocity. This implies that another body of the same size but twice the weight would produce twice the velocity. All the motions we observe around us can then be understood as a balance between the forces that tend to maintain the motion and those that tend to resist it. Stephen Toulmin (1961) argues that by using Aristotle's physics of motion, dealing only with the kind of motion he thought typical, we would discover that his rough proportionalities above 'retain a respected place even in twentieth century physics' (p 51). He goes on to remind us that Stokes' law can be seen as the contemporary version of Aristotle's rough and qualitative ratios connecting gross measures of distances and time. Stokes' law relates the speed of a body moving through a medium with a given viscosity. According to this law the speed of the body will be directly proportional to the force moving it and inversely proportional to the liquid's viscosity:

$$V \propto F/s.$$

This law can easily be tested using a large cylinder containing different liquids, such as water and oil, and then measuring the time of descent of spheres for different densities and radii.

The motion of a projectile presented Aristotle with a really difficult problem. Natural motion, like free fall, required no explanation. And forced motion like the motion of a cart being pulled by a horse can be explained. But what force keeps the projectile in motion after it loses contact with the projector? He thought that the medium somehow

Figure 1. According to Aristotle a cart will come to rest when the horse stops pulling it. Moreover, if objects fall they move through a medium, such as air or water. Aristotle saw a world in which there was *always* a resistance offered to the motion. By committing a slight anachronism we can say that for Aristotle velocity was directly proportional to force and inversely proportional to the resistance of the medium.



provided the necessary force to push the projectile. The paradoxical state of affairs is connected with Aristotle believing that the medium not only sustains the motion but also resists it. Motion in a void was impossible because there was no medium to sustain the motion, and in the absence of resistance the object would eventually move at an infinite speed, clearly an unacceptable solution.

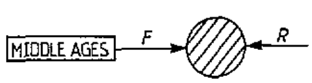
The middle ages

Aristotle's ideas of force and motion were first challenged by John Philoponus (fifth century AD). He rejected the Aristotelian law $V \propto F/R$ and substituted $V \propto F - R$ (see figure 2). Clearly this meant that motion in a void, where the resistance is zero, is possible. Moreover, he argued that it was not air providing the negative power that propelled a projectile, but an *impressed force* that eventually dies out.

John Buridan developed the impetus theory further. He thought that an impressed force on a projectile was permanent unless acted on by resistances or other forces. He defined this impressed force as being proportional to the quantity of matter and the speed. We must be careful, however, not to equate impetus with our concept of momentum. It is not clear, for example, whether he understood impetus as an effect of motion (as we might consider momentum), or as a cause of motion, which would make it similar to force (Franklin 1976).

Buridan did not, however, arrive at a statement of inertia or of the principle of the conservation of momentum. However, a pre-Galilean version of inertia was achieved by Oresme, who was one of his students. Essentially, he argued that it is not possible to detect uniform rectilinear motion. He also though that the Earth rotated, and agreed that the air and water share the motion. The principle of the conservation of linear momentum, however, was not clearly stated until Descartes.

Figure 2. John Philoponus effectively rejected the Aristotelian law $V \propto F/R$ and substituted $V \propto F - R$. This meant that motion in a void, where the resistance is zero, is possible. Moreover, he argued that it was not the air providing the motive power that propelled a projectile, but an *impressed force* that eventually dies out. John Buridan developed the impetus theory further. He thought that an impressed force on a projectile was permanent unless acted on by resistances or other forces. Oresme anticipated Galileo's notion of inertia.



PHILOPONUS (5th century AD)
$V \propto F - R$
Self-expending impressed force

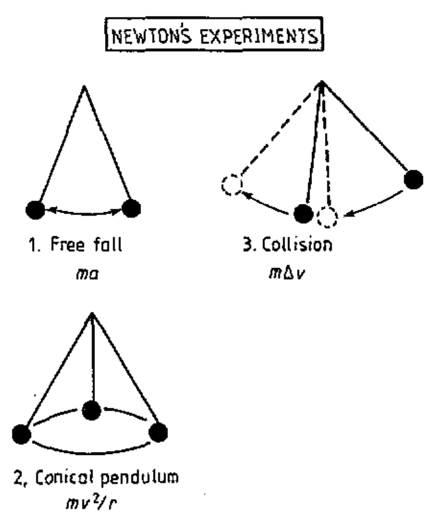
BURIDAN (14th century AD)
Impetus theory
Impressed force = mass × speed
Permanent impetus

ORESME (14th century AD)
Law of inertia?
'It is not possible to detect uniform rectilinear motion.'
'The Earth rotates and air and water share the motion.'

Newton and the central notion of force

Early struggles. Newton's struggle to establish the notion of force as a unifying concept is not a well told story in physics textbooks. Typical accounts of Newton's dynamics get around the challenging problem of giving a historical discussion of how his laws of motion (especially his second law) were formulated. This is achieved by presenting them as

Figure 3. Newton had available three distinct sets of observations that could be connected to three distinct meanings of the notion of force as demonstrated by experiments with the pendulum, the conical pendulum and collision with wooden balls used as pendula. Force had to be defined for each of these cases.



if they were self-evident and came full-blown to the mind of the great man, shortly after the apple fell on his head.

Newton's early struggle to free himself of the idea of impetus (the idea that unassisted motion is sustained by an internal motive force) is discussed in detail by Steinberg *et al* (1990). They argue that the young Newton, like students today, believed in the notion of *impetus*. In addition, students today, like the young Newton, believe in what Steinberg *et al* call *transfer*—the idea that one body may give up some of its force to another during impact.

Eventually, however, Newton transformed the notion of impetus into the concept of inertial mass. His new conception of 'impressed force' as action only signals the complete separation of force and motion. Newton now had the concept of inertial mass and it was possible for him to think of motion without force.

Having arrived at a clear definition of force Newton first turned his attention to the problem of free fall. He wanted to transform Galileo's kinematics into dynamics. It was here, according to Westfall, that he first used one version of his second law, namely that acceleration of a given body is proportional to the impressed force (Westfall 1971, p 357). Free fall provided one sense in which the notion of force as the causal principle of motion was to be understood. The notion of force, however, had to be reconciled with how it was used in two other senses: *force as a meaure of motion* and *force as a measure of change of motion*.

Newton had available three distinct sets of observations that could be connected to three distinct meanings of the notion of force (see figure 3). One set was connected to free fall, as demonstrated by experiments with the pendulum. Another set of observations were connected with the motion of a conical pendulum. Finally, a third set of observations were based on collision with wooden balls used as pendula. Force had to be defined for each of these cases. In the first case we are dealing with constrained free fall, in the second with the force associated with the change of direction of a mass (what it still commonly, but wrongly, referred to as 'centrifugal force'), and in the third we encounter the problem of how to relate the notion of force to impact.

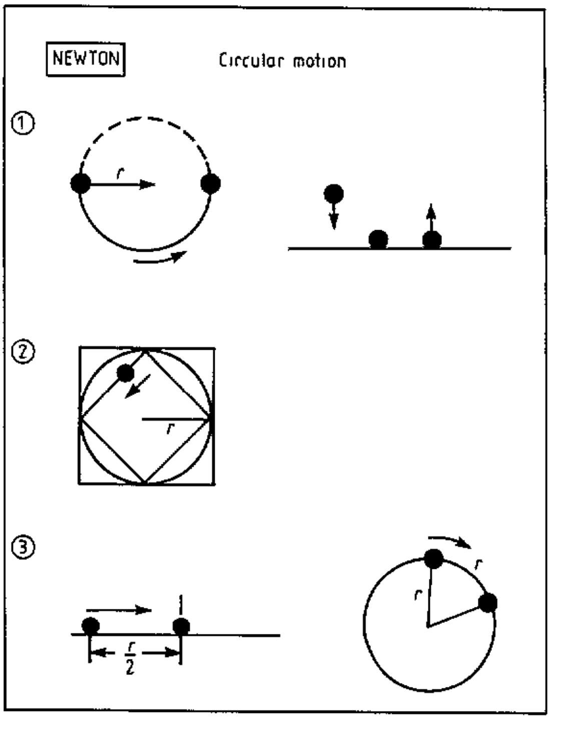
Newton therefore had to find a consistent dynamics to describe these classes of observations. Firstly, accelerated motion in a straight line; secondly, the 'acceleration' connected with a body moving with a constant speed but a changing direction; and finally, the motion involved during collision. The first was associated with the quantity ma, the second with the quantity mv^2/r and the third with the quantity $m\,\Delta v$.

Centrifugal or centripetal? Newton seemed to have had great difficulty in getting rid of the idea of centrifugal (centre-fleeing) force in describing a body in circular motion. I will mention four attempts he made to understand this motion.

In his first attempt to quantify circular motion Newton reasoned that revolution through half a circle is equivalent to a perfectly elastic rebound, which requires a force great enough first to stop a body's forward motion and then to generate an equal motion in the opposite direction. This analogy, however, does not hold and led Newton to dimensionally incommensurable results (see figure 4).

In his second attempt to quantify the force in circular motion Newton imagined a square to be circumscribed around the circle and the ball to follow a path inside it. Taking the component of motion perpendicular to the side, he set down

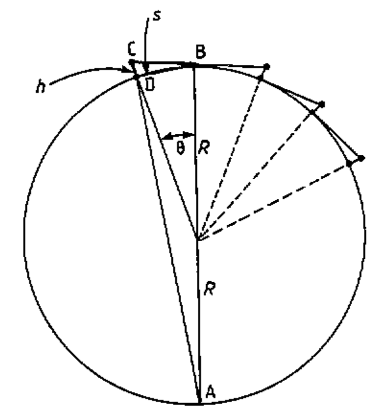
Figure 4. In his first attempt to quantify circular motion Newton reasoned that revolution through half a circle is equivalent to a perfectly elastic rebound. In his second attempt Newton imagined a square to be circumscribed around the circle and the ball to follow a path inside it. In his third attempt he argued that the centrifugal force of a revolving body is such that an equal force, applied to a body of equal mass during the time that the body revolved through one radian, would generate an equal linear velocity in the other body and move it from rest through half the length of a radian.



an expression which compared the force of one impact, in which that component is reversed, to the force of the ball's motion (see figure 4). Newton subsequently realized that if the number of sides of the inscribed and circumscribed polygons is increased, the ratio of force for one circuit continues to equal the ratio of the length of the path to the radius. This approach yields the correct result $a = v^2/r$ (see figure 4).

In this third attempt he argued that the centrifugal force of a revolving body is such that an equal force, applied to a body of equal mass during the time that the body revolved through one radian, would generate an equal linear velocity in the other body and move it from rest through half the length of a radian (see figure 4). This approach also yields the correct result that $a = v^2/r$. Although the last two attempts gave the correct magnitude of the force they *did not suggest the correct direction.* However, until he was able to think of the force as 'centre-seeking', rather than 'centre-fleeing', his dynamics could not be applied to the motion of the planets.

Finally, he managed to derive the formula for 'centrifugal' force in a more economical and elegant way (see figure 5). Here he used the results of Galileo's kinematics of free fall and applied them to the dynamics of a revolving object.



$\triangle ABC = \triangle DCB$ for small θ

$s = vt$

Let $h \simeq \frac{1}{2}a_c t^2$

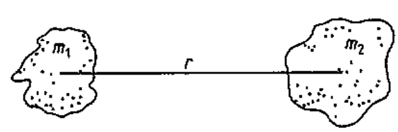
$\Rightarrow \dfrac{h}{s} = \dfrac{s}{2R}$

Substitution gives $a_c = v^2/R$

Figure 5. Newton eventually managed to derive the formula for 'centrifugal' force in a more economical and elegant way. Here he used the results of Galileo's kinematics of free fall and applied them to the dynamics of a revolving object.

A brief note on Newton's third law. The relationship between the second and third laws can be well illustrated by imagining two large masses in close proximity in deep space. Assuming that the gravitational attraction due to each other's mass is the only external force, we can then calculate the acceleration (instantaneous) on the masses. This is a good example for showing the relationship between the second and the third law (see figure 6). In my experience many students in elementary physics do not have a clear understanding of the relationship between the two.

NEWTON'S SECOND AND THIRD LAWS



- What are the forces on m_1 and m_2?
- What is the instantaneous acceleration of m_1 and m_2?

Figure 6. The relationship between the second and third laws can be seen here when students consider the effect that two large masses have on each other in deep space.

Force and the idea of absolute motion. Newtonian laws of motion are valid only in non-accelerating or inertial frames of reference. Newton spent a great deal of effort trying to explain this privileged status of inertial frames and postulated the existence of absolute space and time. He thought that inertial forces provide a clear indication of absolute motion.

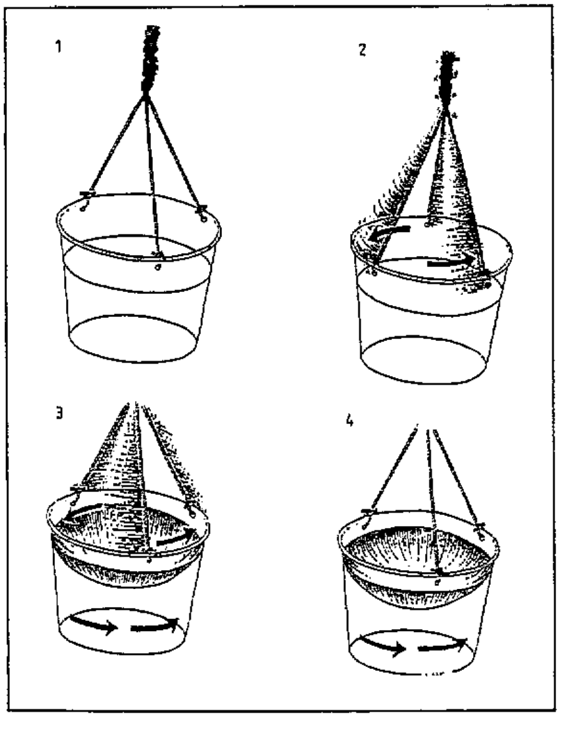
To illustrate how absolute motion could be determined he presented two thought experiments: his famous *bucket experiment* and the experiment involving the *rotation of two globes in an immense void* (see figures 7 and 8). These thought experiments show that the relative rotation of the water with respect to the bucket is not responsible for the centrifugal forces. Moreover, Newton argued that the tension in the cord in figure 8, due to centrifugal forces, would be registered even in a void where no other masses existed.

Perhaps the cause is the Earth itself or the Solar System? But Foucault's pendulum experiments show that the cause is to be found outside the Earth. Moreover, the orbits of planets are sustained by virtue of 'centrifugal' forces. It seems then that the phenomenon of 'centrifugal' forces is universal and cannot be due to interaction (Born 1965, p 84).

Newton's notion of force and the continental physicists

It is noteworthy that continental physicists, notably Leibnitz and Huygens during Newton's lifetime, and later Lagrange, did not follow Newton in trying to establish a coherent dynamics based on a unifying concept of force. Instead, they emphasized the role that conservation laws played in collisions. Leibnitz replaced the 'momentum' of Newton by the 'kinetic energy' and the 'force' of Newton by the 'work of the force'. Later this 'work of the force' was replaced by a still more basic quantity, namely the 'work function'. Leibnitz became the originator of 'analytical mechanics', which based the entire study of equilibrium and motion on the scalar quantities of kinetic and potential energy. Newton himself, however, stood aside from the continental controversy as to whether mv or mv^2 was the best measure of the 'quantity of motion'.

Figure 7. According to Newton the thought experiment with the bucket shows that the relative rotation of the water with respect to the bucket is not responsible for the centrifugal forces. At stage 2 as well as stage 4 the bucket and the water are in motion relative to each other. However, in the first case the water has a level surface and in the latter case a concave surface. To explain this, Newton felt compelled to posit the notion of absolute motion.



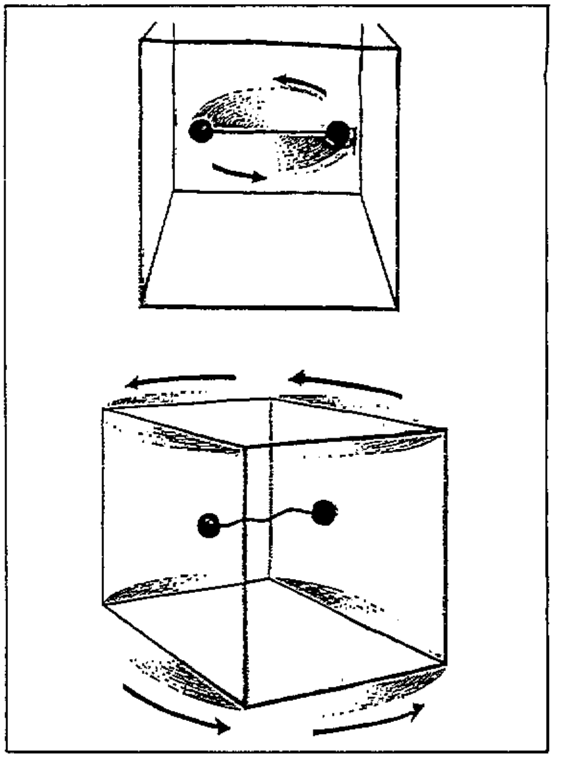


Figure 8. Newton proposed a second thought experiment to clinch the argument for absolute motion. He argued that the tension in the cord, due to centrifugal forces, would be registered even in a void where no other masses existed.

One could argue that it may indeed have been fortuitous that Newton pursued his particular line of reasoning. His conception lends itself easily to considerations of gravitation as a situation of natural acceleration. Indeed, the crucial identification of his physics is that of the notion of 'forcing' and 'accelerating', where 'accelerating' is understood as a mass changing its velocity in some law-like way. The continental assimilation of 'forcing' into 'moving' meant that no clear account of gravitation could be given.

Mach and absolute motion

Ernst Mach criticized Newton's argument for absolute motion. He began by arguing that mechanical experience can never teach us anything about absolute space. We can only measure relative motions and therefore only they are physically real. He concluded that Newton's idea of absolute space therefore must be illusory. Indeed, Newton's whole argument seems to depend on whether or not we admit that if the stars (today we would say 'the whole universe') were to rotate about the Earth no flattening and no decrease of gravity at the equator would occur (Born 1965, p 84).

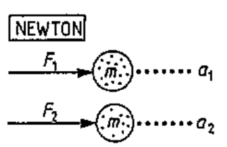
Mach argued that mass is not endowed with the intrinsic property called inertia. Rather, he thought that inertia is the feature of the effect of the entire mass in the universe understood as a closed system.

Specifically, Mach showed that one could equally well derive the Newtonian equations of motion from a Galilean relativistic point of view (Sachs 1974, pp 101–3). This argument is given in a simplified form in figure 9.

Mach also speculated about the local and global material interactions that might be responsible for

Figure 9. Mach showed that one could equally well derive the Newtonian equations of motion from a Galilean relativistic point of view.

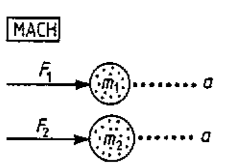
Mach's argument



$$\Rightarrow \frac{F_1}{F_2} = \frac{a_1}{a_2} \qquad \text{(observable)}$$

or $F = ka$, when $k = m$.

⇒ Inertial mass is a measure of a particular intrinsic property of matter.



$$\Rightarrow \frac{F_1}{F_2} = \frac{m_1}{m_2} \qquad \text{(observable)}$$

or $m = KF$

when K $(= m_2/F_2$, say) may be used as a standard for comparison, with applications to all matter.

m is now seen as a measure of the *coupling* between this matter and all of the other matter of a *closed system*.

inertial effects. For example, he wanted to know what would be the local inertial effects of a very large rotating bucket on 'stationary' water within due to the resulting relative rotation between bucket and water.

The interpretation of inertial mass as a manifestation of a closed system was later (1918) named by Einstein the 'Mach principle' (Sachs 1974, p 103).

Einstein banishes the notion of force

Einstein based his general theory of relativity (GTR) on two postulates: (i) the principle of relativity (the laws of nature must be given in terms of continuous field variables) and (ii) Mach's principle (the fundamental description of any realistic physical system must be closed) (Sachs 1974, p 108). Einstein began his follow-up studies of his special theory of relativity (STR) with the question: why should the principle of relativity apply only to frames of reference that are moving with a constant velocity relative to each other? If there is a law of nature implying that A must accelerate relative to B, because some physical cause creates this effect, then there should be an expression of the identical law, leading to the conclusion that B is accelerating relative to A, again predicted by a corresponding cause–effect relation, but from B's frame of reference, rather than from A's (Sachs 1974, p 108).

Non-uniform motion is, of course, the only kind of motion that can actually be experienced by matter, when matter interacts with other matter. This is so because matter interacts due to a force (by definition), causing mutual transfer of energy and momentum. But force is the cause of non-uniform motion. Therefore, Einstein reasoned, STR must be a limiting case of the theory of relativity (Sachs 1974, p 108).

An entirely new formulation of the law of inertia came to Einstein in a celebrated thought experiment, the 'happiest thought of my life', as he later recalled (Holton 1979). Here is a modified version of his thought experiment.

We can imagine an observer in a giant elevator in deep space that is attached to a cable pulling the elevator with a force that accelerates it at 1g (about 10 m s^{-2}) (see figure 10). Similarly imagine an identical elevator simply hovering over the surface of the Earth. We can now argue that the observer in the accelerating elevator and the observer in the Earth-elevator could not find an experiment to differentiate between their physics. Even a laser beam would bend the same amount in each case (if this could be measured).

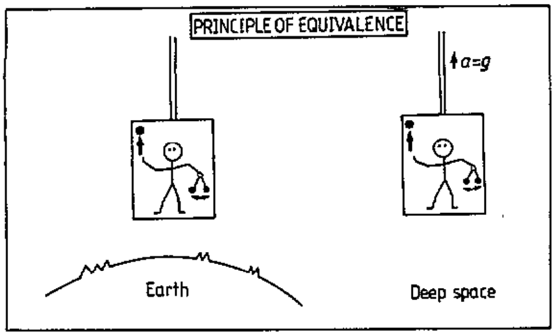


Figure 10. We can imagine an observer in a giant elevator in deep space that is attached to a cable pulling the elevator with a force that accelerats it at 1 *g*. Similarly imagine an identical elevator simply hovering over the surface of the Earth. We can now argue that the observer in the accelerating elevator and the observer in the Earth-elevator could not find an experiment to differentiate between their physics. Einstein therefore concluded that the effects of gravitation and those due to acceleration cannot be distinguished.

Einstein therefore concluded that the effects of gravitation and those due to acceleration cannot be distinguished. Newtonian mechanics distinguishes between the motion of a body that is inertial (subject to no forces) and the motion of a body subject to the action of gravity. The former is rectilinear and uniform in an inertial system; the latter occurs in curvilinear paths and is non-uniform (Born 1965, p 315). The principle of equivalence, however, does not allow this distinction. Einstein's mandate now was to state the law of inertial motion in the generalized sense. The solution of this problem banishes both the notion of absolute space *and* force and gives us a theory of gravitation.

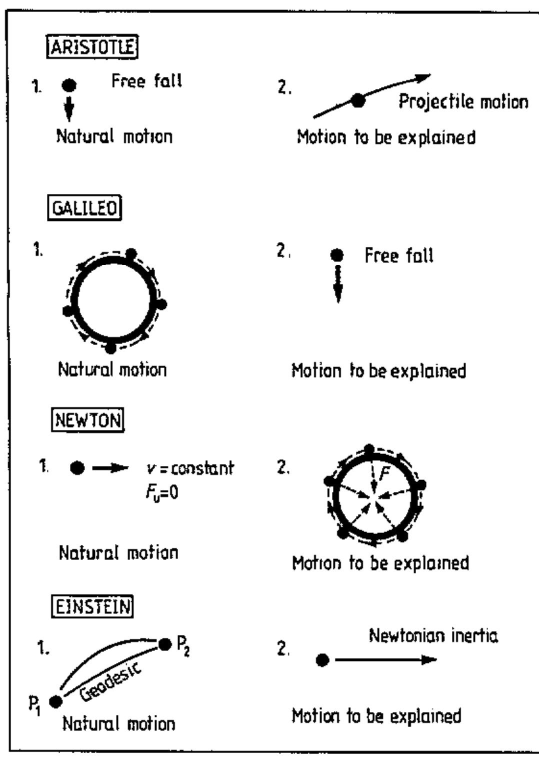
In Newton's theory the symbol F in $F=ma$ refers to the cause of the acceleration of the body. Force, then, is an external agent that acts on matter with an inertial mass m, causing it to accelerate at the rate a. In the GTR, however, there is no external force. Indeed, Einstein was able to derive Newton's equation $F=ma$ from purely geometric considerations. He saw the possibility that all 'external' forces may be only apparent—that the 'effect' of other matter may be representable by a generalization of the geometry of space–time that describes the motions.

Conclusion

In order to relate the story of force from Aristotle to Einstein to ordinary experience one might present a picture of the relationship between force and motion based on Toulmin's 'ideals of natural order' (1961). According to this picture, for Aristotle natural motion was the free fall of an object. For Galileo it was understood as the unimpeded circumnavigation of the Earth by a ship (gravity still acted on the ship but did not resist the motion), and for Newton natural motion was the constant velocity motion of a body in deep space (no external forces). Einstein banished the notion of force and described the motion of a free particle in non-Euclidean space. For him natural motion became the path of a free particle along a geodesic—the path of minimal separation (see figure 11).

I have presented 'A brief history of force' in several high school classes as well as to physics educators at two universities, along the lines discussed in this paper. The equipment required for the presentation is easily obtained. It consists of two long pendula (about 3 m) with wooden balls, two large graduated cylinders (one filled with

Figure 11. For Aristotle natural motion was the free fall of an object. For Galileo it was understood as the unimpeded circumnavigation of the Earth by a ship, and for Newton natural motion was the constant velocity motion of a body in deep space (no external forces). Einstein banished the notion of force and described the motion of a free particle in non-Euclidean space. For him natural motion became the path of a free particle along a geodesic—the path of minimal separation.



water, the other with oil) with two small spheres of the same radius but different masses, a bucket filled with water and attached to a ceiling by a rope, and a simple 'accelerometer' assembly to illustrate Einstein's equivalence principle. The latter demonstration is especially puzzling since the motion of the two spheres when acceleration occurs is counter-intuitive.

One should begin by challenging students to articulate their own ideas in response to Aristotle, Buridan, Galileo, Newton, Mach and Einstein. The success of this kind of historical presentation depends on the knowledge of the instructor and his/her ability to assume the roles of these key figures with conviction, i.e. the ability to defend their views. I have found this approach very exciting, motivational, with good potential for examining the level of the students' conceptualization of the notion of force.

In the classroom physics teachers often present the Euler version of Newton's second law, namely $F=ma$, after only a cursory discussion of the relationship between unbalanced force, mass and acceleration. Students then memorize algorithms for type problems without a clear understanding of the history and subtlety of the concept of force (or of acceleration and mass) in this formulation. If experimental work is done at all, it is often accomplished by having an 'accelerating' mass attached to a string over a pulley connected to a dynamic cart on the table. The student often does not understand the procedure for separating the variables and will generally not distinguish between gravitational and inertial masses. The student then commits to memory a number of algorithms for solving representative (type) problems in the textbook.

David Hestenes *et al* (1992), referring to the teaching of elementary dynamics, say that 'since the students have evidently not learned the most basic Newtonian concepts, they must have failed to comprehend most of the material in the course' (p 141). We now have overwhelming evidence that even after successful completion of an elementary course in physics the central notion of force for most students is surrounded with misconceptions (Hestenes *et al* 1992).

Of course, I am not saying that the conventional textbook-centred physics teaching does not produce students who are well prepared for university work in science. Clearly it does: one could argue that we have successfully trained physicists for generations using this trusted approach. But we may have accomplished that at a price, losing many potential physicists and giving a lower quality education to the student who does not go on to specialize in physics. What I am trying to promote then is the training and educating of competent and enthusiastic physics teachers to present major concepts of physics in a story format that is rich in diverse connections for all physics students.

In incorporating the story of force into an introductory physics course, one would present segments of the story as the course progressed. A periodic injection of historical context by a knowledgeable teacher into the conventional textbook-centred physics classroom might not produce more physicists but it would almost certainly enhance the richness of presenting the exciting ideas of physics. Who knows, by using a contextual setting of big ideas (see Stinner and Williams 1993) our future physicists may gain a deeper understanding of their craft and become more exciting teachers themselves. It might even increase the physics literacy of the general population. This is a modest expectation and a worthy goal.

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